ISSN 1061-9208, Russian Journal of Mathematical Physics, 2015, Vol. 22, No. 4, pp. 437–443. © Pleiades Publishing, Ltd., 2015.

The Sphere in $\mathbf{C}^2$ as a Model Surface for Degenerate Hypersurfaces in $\mathbf{C}^3$

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Received January 3, 2015

Abstract. In the paper, an unexpected correspondence between the automorphisms of 5D real uniformly 2-nondegenerate hypersurfaces of the space $\mathbf{C}^3$ and the automorphisms of the 3D hypersphere in $\mathbf{C}^2$ is constructed. In a certain sense, the 3D hypersphere, which is, as is known, a model surface for the class of nondegenerate 3D hypersurfaces in $\mathbf{C}^2$, has this status also with respect to the above class of 5D hypersurfaces in $\mathbf{C}^3$.

DOI 10.1134/S1061920815040020

1. INTRODUCTION

If (z, w) are the coordinates of the space $\mathbf{C}^2$, then the local equation of a Levi-nondegenerate real analytic hypersurface has (after an analytic simplification) the form $\Im w = |z|^2 + O(3)$ ($O(m)$ stands for the terms of the series of degree at least m). If (z, ζ, w) are the coordinates of the space $\mathbf{C}^3$, then the local equation of the hypersurface whose Levi form at the origin is of rank one has (after an analytic simplification) quite the same form $\Im w = |z|^2 + O(3)$. Here the equation $\Im w = |z|^2$ in $\mathbf{C}^2$ defines a 3D hypersurface projectively equivalent to the standard sphere $|z|^2 + |w|^2 = 1$, and the group of holomorphic automorphisms of this hypersurface is isomorphic to the 8D classical group $SU(2, 1)$. At the same time, the 5D hypersurface defined by the same equation in $\mathbf{C}^3$ has the structure of the direct product of the sphere in $\mathbf{C}^2$ by the complex line and its automorphism group is infinite-dimensional. The hypersurface of the form

$$\Im w = |z|^2 + O(3)$$

in $\mathbf{C}^3$ has rank one at the origin, and the rank in a neighborhood of the origin is not less than one. If the rank of a hypersurface is equal to one in a full neighborhood of the origin, then we say that our *hypersurface is uniformly Levi-degenerate*. The condition for the validity of this property for our hypersurface is the condition that the determinant of the matrix of the Levi form, which is a Hermitian matrix of size (2×2), vanishes. Further, the following dichotomy occurs. If there is a holomorphic vector field L such that $L + \bar{L}$ occurs in the *complex tangent* to the hypersurface at every point, then the surface is said to be *holomorphically degenerate* and again has (in certain local holomorphic coordinates) the form of direct product of a hypersurface in $\mathbf{C}^2$ and the complex line. The automorphism group is infinite-dimensional in this case again. In the opposite case, the hypersurface satisfies a nondegeneracy condition, which is referred to as 2-*nondegeneracy* (see [4]). We say that *every surface of this kind is uniformly* 2-*nondegenerate*, and this surface is the main object of our investigation. For the subsequent presentation, it is more convenient to speak about the weights of monomials rather than on their degrees. We define the weights of the monomials by the following choice of the weights of the variables:

$$[z] = [\bar{z}] = [\zeta] = [\bar{\zeta}] = 1, \qquad [w] = [\bar{w}] = 2.$$

[1]The research was financially supported by the Russian Foundation for Basic Research under grants nos. 14-01-00709-A and 13-01-12417-OFI-M-2013.

[2]Supported by the Austrian Science Fund (FWF).

We treat now the notation $O(m)$ as the sum of monomials whose weights are greater than or equal to m. It can readily be seen that a uniformly 2-nondegenerate hypersurface in $\mathbf{C}^3$ has an equation of the form

$$\Im w = |z|^2 + 2\Re(z^2\bar{\zeta}) + O(4)$$

(see [6]). As was proved in [1], the dimension of the stabilizer of the origin in the group of holomorphic automorphisms of this hypersurface does not exceed six, and the dimension of the entire group does not exceed 11. Combining this result with the known classification of Fels and Kaup [7] of the homogeneous 2-nondegenerate hypersurfaces, one can readily improve the bound for the dimension of the full group to 10. This result is exact, because 10 is the dimension of the light cone

$$(\Im w)^2 = (\Im z)^2 + (\Im \zeta)^2.$$

The dimension of the stabilizer of a point of the light cone is equal to five (see [8]), which coincides wonderfully with the dimension of the stabilizer of a point of the sphere in $\mathbf{C}^2$. The present paper contains an explanation of this coincidence (see Theorem 5 below). Note that an exact bound for the dimension of a stabilizer was obtained in [9], where the authors have proved that the light cone has the largest stabilizer in the class of uniformly 2-nondegenerate hypersurfaces.

1. HOMOLOGICAL OPERATOR, THE KERNEL, AND THE NORMALIZATION

Let Γ_ξ be the germ of a real analytic uniformly 2-nondegenerate hypersurface of 3D complex space. Then one can choose a coordinate system $(z \in C,\ \zeta \in C,\ w = u+iv \in C)$ in a neighborhood of ξ in such a way that the equation Γ_ξ becomes

$$v = z\bar{z} + \Re(z^2\bar{\zeta}) + F_4 + F_5 + \cdots \tag{1}$$

Here we assign the weights to the variables as follows:

$$[z] = [\bar{z}] = [\zeta] = [\bar{\zeta}] = 1, \qquad [u] = 2.$$

Let us apply the standard apparatus in the style of Poincaré (formal series and the homological equation) to this class of hypersurfaces. Let

$$z \to z + f_3 + \cdots, \quad \zeta \to \zeta + h_2 + \cdots, \quad w \to w + g_4 + \cdots, \tag{2}$$

takes Γ_0 onto another germ of the same kind, $\tilde{\Gamma}_0$, which is given by the equation

$$v = z\bar{z} + 2\Re(z^2\bar{\zeta}) + \tilde{F}_4 + \tilde{F}_5 + \cdots \tag{3}$$

In this case, writing out the relation thus occurring and distinguishing the component of weight m in this relation, we obtain

$$\Re(ig_m(z,\zeta,w) + 2\bar{z}f_{m-1}(z,\zeta,w) + 2\bar{z}^2 h_{m-1}(z,\zeta,w)) = \tilde{F}_m(z,\bar{z},\zeta,\bar{\zeta},u) - F_m(z,\bar{z},\zeta,\bar{\zeta},u) + \cdots,$$

where $w = u + i|z|^2$, and this w is of weight 2, and the dots stand for the terms depending on $g_\mu, f_{\mu-1}, h_{\mu-2}, F_\mu, \tilde{F}_\mu$ for $\mu < m$.

Let us evaluate the kernel of the homological operator

$$\mathcal{L}(g,f,h) = \Re(ig(z,\zeta,u+i|z|^2) + 2\bar{z}f(z,\zeta,u+i|z|^2) + 2\bar{z}^2 h(z,\zeta,u+i|z|^2)) = 0. \tag{4}$$

Introduce the notation

$$\begin{gathered} f(0,0,u) = a(u), \quad g(0,0,u) = b(u), \quad h(0,0,u) = c(u), \\ \frac{\partial f}{\partial z}(0,0,u) = d(u), \quad \frac{\partial h}{\partial z}(0,0,u) = e(u), \quad \frac{\partial^2 h}{\partial z^2}(0,0,u) = m(u). \end{gathered} \tag{5}$$

Writing $\bar{z} = \bar{\zeta} = 0$ in (4), we obtain

$$g(z, \zeta, u) = b(u) + 2i\bar{a}(u)z + 2I\bar{c}(u)z^2. \tag{6}$$

For $z = 0$, we obtain $\Im b(u) = 0$. Substitute the expression thus obtained into (4), differentiate with respect to $\bar{z}$, and set $\bar{z} = \bar{\zeta} = 0$. We obtain

$$f(z, \zeta, u) = a(u) + d(u)z + (2i\bar{a}'(u) - \bar{e})z^2 + 2i\bar{c}(u)z^3. \tag{7}$$

Here $b'(u) = 2\Re d(u)$. Substitute the expression thus obtained into (4), differentiate with respect to $\bar{z}$ twice, and set $\bar{z} = \bar{\zeta} = 0$. We obtain

$$h(z, \zeta, u) = c(u) + ze(u) + \frac{1}{2}m(u)z^2 + (2\bar{a}''(u) + 2i\bar{e}'(u))z^3 + 2\bar{c}''(u)z^4. \tag{8}$$

Here $2\Im d'(u) = \Re m(u)$. Substitute the expression thus obtained into (4) and write out the form of several monomials entering (4) after three our substitutions,

$$\begin{aligned}
&\Big(\frac{1}{3}b'''(u) - \bar{d}''(u) - d''(u) - i\bar{m}'(u) + im'(u) - i\bar{m}'\Big)z^3\bar{z}^3,\\
&\Big(-4\bar{e}'(u) + \frac{8}{3}i\bar{a}'''(u)\Big)z^4\bar{z}^3,\\
&\frac{8}{3}i\bar{c}'''(u)z^5\bar{z}^3,\\
&\Big(\frac{1}{3}i\bar{d}'''(u) - \frac{1}{3}id'''(u) - \frac{1}{2}m''(u) - \frac{1}{2}\bar{m}''(u)\Big)z^4\bar{z}^4,\\
&\Big(-\frac{4}{3}a^{IV}(u) + \frac{4}{3}ie'''(u)\Big)z^5\bar{z}^4,\\
&\Big(\frac{1}{12}d^{IV}(u) + \frac{1}{12}\bar{d}^{IV}(u) - \frac{1}{60}b^{V}(u) + \frac{1}{6}i\bar{m}'''(u) - \frac{1}{6}im'''(u)\Big)z^5\bar{z}^5.
\end{aligned} \tag{9}$$

If all these summands are equal to zero, then all six functions of one variable, (a, b, c, d, e, m), are polynomials with the following bounds for the degrees: $(3, 4, 2, 3, 2, 2)$. We obtain finally the following description of the kernel of the operator $\mathcal{L}$.

Lemma 1.

(*a*) *The kernel of* $\mathcal{L}$ *is the* $27D$ *space which consists of the polynomials of the form*

$$\begin{aligned}
g &= b_0 + 2\Re d_0 u + \Re d_1 u^2 + \frac{2}{3}\Re d_2 u^3 + \frac{1}{2}d_3 u^4 + 2i(\bar{a}_0 + \bar{a}_1 u + \bar{a}_2 u^2\\
&\quad + \bar{a}_3 u^3)z + 2i(\bar{c}_0 + \bar{c}_1 u + \bar{c}_2 u^2)z^2,\\
f &= a_0 + a_1 u + a_2 u^2 + a_3 u^3 + (d_0 + d_1 u + d_2 u^2 + d_3 u^3)z\\
&\quad + (2i\bar{a}_1 + 4i\bar{a}_2 u + 4i\bar{a}_3 u^2 - \bar{e}_0 - \bar{e}_1 u)z^2 + (2i\bar{c}_1 + 4i\bar{c}_2 u)z^3,\\
h &= 4\bar{c}_2 z^4 + (4\bar{a}_2 + 4\bar{a}_3 u + 2i\bar{e}_1)z^3\\
&\quad + (\Im d_1 + \frac{1}{2}im_0 + 2u\Im d_2 - \frac{2}{3}iu\Re d_2 - id_3 u^2)z^2 + z(e_0 + e_1 u - 2ia_3 u^2) + c_0 + c_1 u + c_2 u^2.
\end{aligned} \tag{10}$$

Here $(a_0, a_1, a_2, a_3, c_0, c_1, c_2, d_0, d_1, d_2, e_0, e_1)$ *are complex parameters and* (b_0, d_3, m_0) *are real ones.*

(*b*) *If one considers the expansion of the kernel with respect to the weight components, then the leading component is* (g_8, f_7, h_6).

The real power series F in the variables $(z, \bar{z}, \zeta, \bar{\zeta}, u)$ can be represented in the form

$$\sum c_{\alpha,\beta,\gamma,\delta}(u) z^\alpha \bar{z}^\beta \zeta^\gamma \bar{\zeta}^\delta.$$

The following components were used to evaluate the kernel of the operator:

$$(\alpha, 0, \gamma, 0), \quad (\alpha, 1, \gamma, 0), \quad (\alpha, 2, \gamma, 0)$$

for all values of α and β, and also

$$(3,3,0,0),\ (4,3,0,0),\ (5,3,0,0),\ (4,4,0,0),\ (5,4,0,0), \quad \text{and} \quad (5,5,0,0).$$

Define the direct decomposition of the space of real power series $\mathcal{F}$ into the direct sum of two subspaces $\mathcal{R}$ and $\mathcal{N}$, where $\mathcal{R}$ are the sums of monomials of the above form and the sums conjugate to them and $\mathcal{N}$ are the real series composed of the monomials which do not enter $\mathcal{R}$. Repeating the same calculations for the nonhomogeneous equations, we obtain the following assertion.

Lemma 2. *The linear nonhomogeneous equation* $\mathcal{L}(g, f, h) = T \ mod \ \ \mathcal{N}$ *is solvable for an arbitrary right-hand side* T.

Using relation (4) for the recurrent evaluation of the consecutive triples (g_m, f_{m-1}, h_{m-2}) and, using the two lemmas proved above we obtain the following proposition.

Proposition 3.

(*a*) *The equation of a hypersurface of the form* (1) *can be reduced by a formal change of variables to a formal normal form*

$$\Im w = |z|^2 + 2\Re(z^2\bar{\zeta}) + O(4) = |z|^2 + 2\Re(z^2\bar{\zeta}) + \sum N_{\alpha,\beta,\gamma,\delta}(u) z^\alpha \bar{z}^\beta \zeta^\gamma \bar{\zeta}^\delta, \tag{11}$$

where

$$N_{\alpha,0\gamma,0}(u) = N_{\alpha,1\gamma,0}(u) = N_{\alpha,2\gamma,0}(u) = 0$$

for all values of α *and* γ, *and also*

$$N_{3,3,0,0}(u) = N_{4,3,0,0}(u) = N_{5,3,0,0}(u) = N_{4,4,0,0}(u) = N_{5,4,0,0}(u) = N_{5,5,0,0}(u) = 0.$$

(*b*) *If there are two mappings of the form*

$$w \to w + O(9), \quad z \to z + O(8), \quad \zeta \to \zeta + O(7)$$

of one hypersurface of the form (1) *onto another hypersurface of this kind* (*it is assumed that both the surfaces are chosen*), *then these mappings coincide. In particular, if this is a mapping of a hypersurface onto itself, then it is the identity mapping.*

Note that, if we need a normalization of a finite segment of the series, then this normalization can be achieved by a polynomial change of variable.

2. CONDITION OF UNIFORM 2-NONDEGENERACY

Let Γ_ξ be the germ of a real analytic hypersurface of 3D complex space whose Levi form is degenerate and has a nonzero rank 1 at a point ξ. Then one can choose a coordinate system $(z \in C,\ \zeta \in C,\ w = u + iv \in C)$ in a neighborhood of ξ in such a way that the equation Γ_ξ becomes

$$v = z\bar{z} + \text{terms of degree at least } 3 = z\bar{z} + F(z, \bar{z}, \zeta, \bar{\zeta}, 2u). \tag{12}$$

Suppose now that the surface is uniformly 2-nondegenerate; let us study the consequences of this condition. The equation of the tangent is of the form

$$dv = 2\Re(F_z dz + F_\zeta d\zeta) + F_u du.$$

The equation of the complex tangent is

$$dw = (F_z dz + F_\zeta d\zeta)/(1 - iF_u).$$

The Levi form is the restriction of the complex Hessian $\partial\bar\partial(|z|^2+F)$ to the complex tangent, i.e., an Hermitian form on the space of variables (z,ζ) to which an Hermitian matrix of size 2×2 corresponds, namely, $\mathcal{L}(F)$. Since $F=O(3)$, it follows that this form has rank one at the origin, and therefore, the condition of 2-nondegeneracy is reduced to the condition

$$\det\mathcal{L}(F)=0. \tag{13}$$

Assume that the lower terms (up to the weight 9) of the equation of the hypersurface are represented in our normal form, i.e.,

$$v=|z|^2+2\Re(z^2\bar\zeta)+N_4+N_5+N_6+N_7+N_8+N_9+O(10). \tag{14}$$

It is clear that the $(m-2)$nd component of (13) does not depend on the components of the equation whose weights exceed m. Distinguishing in (13) the components of weights 2, 3, 4, 5, 6, and 7 and equating them to zero, we obtain conditions on the form of the terms N_4, N_5, N_6, N_7, N_8, N_9. The computer-aided calculations from [3] so that

$$N_4=4|z|^2|\zeta|^2,\qquad N_5=2\Re(4z^2\zeta\bar\zeta^2+\beta_1z^3\bar z\bar\zeta+\beta_2z^4\bar\zeta), \tag{15}$$

where β_1 and β_2 real. The components of the subsequent weights have a specific form; they depend on an increasing family of parameters [3].

3. LOWER-ORDER COMPONENTS OF AN AUTOMORPHISM

Our next objective is the investigation of the lower-order components of an automorphism of a hypersurface of the form

$$v=z\bar z+2\Re(z^2\bar\zeta)+4|z|^2|\zeta|^2+N_5+N_6+N_7+N_8+\cdots=N(z,\bar z,\zeta,\bar\zeta,u) \tag{16}$$

represented by an equation in normal form, taking into account the restrictions obtained from the condition of uniform Levi-degeneracy. We are interested in the 8-jet of an automorphism preserving the origin, namely,

$$\begin{aligned} w&\to g_1+g_2+g_3+g_4+g_5+g_6+g_7+g_8+O(9)=g(z,\zeta,w),\\ z&\to f_1+f_2+f_3+f_4+f_5+f_6+f_7+O(8)=f(z,\zeta,w),\\ \zeta&\to h_1+h_2+h_3+h_4+h_5+h_6+O(7)=h(z,\zeta,w). \end{aligned} \tag{17}$$

The fact that this mapping is an automorphism means precisely that

$$\begin{aligned}\Phi(z,\bar z,\zeta,\bar\zeta,u)&=-\Im g+N(f,\bar f,h,\bar h,\Re g)\\ &=-\Im g+f\bar f+2\Re(f^2\bar h)+4|f|^2|h|^2+N_5+N_6+N_7+N_8+\cdots=0\ \text{ for }\ w=u+iN.\end{aligned} \tag{18}$$

This relation can be expanded in a sum of weight components, $\sum\Phi_m$. It is clear that the component of weight m does not depend on $(g_\mu,f_{\mu-1},h_{\mu-2})$ for $\mu>m$. Let us single out the lower-order components [3].

Weight 1. $\Phi_1=-\Im g_1$, which implies that $g_1=0$.

Weight 2. $\Phi_2=-\Im g_2+|f_1|^2$, which implies that

$$g_2=|\lambda|^2w,\quad f_1=\lambda z,$$

where λ is a nonzero complex number.

Weight 3.

$$g_3=|\lambda|^2 2i\bar a zw,\qquad f_2=\lambda(2i\bar a-b)z^2+aw),\qquad h_1=\frac{\lambda}{\bar\lambda}(\zeta+bz),$$

where a and b are complex parameters.

Weight 4. We see that $b = -ia$, and also

$$g_4 = |\lambda|^2((A_1 w - 2\bar{a}^2)z^2 w + (r + i|a|^2)w^2),$$
$$f_3 = \lambda(A_1 z^3 + zw(r + iA_2) - 2\bar{a}\zeta w),$$
$$h_2 = \frac{\lambda}{\bar{\lambda}}\left(z^2(A_2 - 3/2|a|^2 + iA_3) + 2i\bar{a}zp + \frac{1}{2}i\bar{A}_1 w\right),$$

where A_1 is a complex parameter and A_2 and A_3 are real ones. We carry out the subsequent evaluations under the assumption that the values of the parameters (λ, a, r) coincide with their values for the identity mapping, namely,

$$\lambda = 1, \quad a = 0, \quad r = 0.$$

Weight 5. We obtain $A_1 = A_2 = A_3 = 0$, which implies that

$$g_1 + g_2 + g_3 + g_4 = w, \quad f_1 + f_2 + f_3 = z, \quad h_1 + h_2 = \zeta,$$

and we also have

$$g_5 = 2i\bar{B}zw^2, \quad f_4 = Bw^2, \quad h_3 = -4\bar{B}z^3 - 4iBzw,$$

where B is a complex parameter.

Weight 6. We obtain $B = 0$, which implies that $g_5 = f_4 = h_3 = 0$; we also have

$$g_6 = C_1 z^2 w^2, \quad f_5 = 2C_1 z^3 w + iC_2 zw^2, \quad h_4 = -2iC_1 z^4 + 2C_2 z^2 w + \frac{i}{2}\bar{C}_1 w^2,$$

where C_1 is a complex parameter and C_2 is a real one.

Weight 7. We obtain $C_1 = C_2 = 0$, which implies that $g_6 = f_5 = h_4 = 0$; we also have

$$g_7 = Dzw^3, \quad f_6 = 2Dz^2 w^2 + \frac{i}{2}\bar{D}w^3, \quad h_5 = -2iDz^3 w + \bar{D}zw^2,$$

where D is a complex parameter.

Weight 8. We obtain $D = 0$, which implies that $g_7 = f_6 = h_5 = 0$; we also have

$$g_8 = Ew^4, \quad f_7 = 2Ezw^3, \quad h_6 = -2iEz^2 w^2,$$

where E is a real parameter.

Weight 9. We obtain $E = 0$, which implies that $g_8 = f_7 = h_6 = 0$.

Finally, as an immediate consequence of Proposition (3.b), we obtain the following proposition.

Proposition 4. *If $z \to f(z,\zeta,w)$, $\zeta \to h(z,\zeta,w)$, $w \to g(z,\zeta,w)$ is an automorphism of a uniformly 2-nondegenerate hypersurfaces of the form (1) and if*

$$\frac{\partial f}{\partial z}(0,0,0) = 1, \qquad \frac{\partial f}{\partial w}(0,0,0) = 0, \qquad \Re\left(\frac{\partial^2 g}{\partial w^2}(0,0,0)\right) = 0,$$

then this automorphism is the identity mapping, i.e., $f = z$, $h = \zeta$, $g = w$.

Recall some well-known facts concerning nondegenerate hypersurfaces in C^2. The hypersurface $Q = \{v = |z|^2\}$ in C^2 is the projective image of the standard sphere $S = \{|z|^2 + |w|^2 = 1\}$. The hypersurface Q is holomorphically (to be more precise, affine) homogeneous, and the stabilizer of the origin (of the point $\xi = (0,0)$) of $\mathrm{Aut}_\xi\, Q$ consists of the projective transformations of the form

$$z \to \frac{\lambda(z + aw)}{1 - \delta}, \qquad w \to \frac{|\lambda|^2 w}{1 - \delta}, \qquad \delta = 2i\bar{a}z + (r + i|a|^2)w), \tag{19}$$

where $\lambda \in C^*$, $a \in C$, $r \in R$. Denote this mapping by $\chi_{\lambda ar}$.

If (F, G) are the coordinate functions of this mapping, then

$$\lambda = F'_z(0,0), \qquad a = \frac{F'_w(0,0)}{F'_z(0,0)}, \qquad r = \frac{\Re G''_{ww}(0,0)}{|F'_z(0,0)|^2}. \tag{20}$$

If we use the weights $[z] = [\bar{z}] = 1$ and $[w] = [\bar{w}] = 2$, then every Levi-nondegenerate hypersurface in C^2 has a local equation of the form $v = |z|^2 + O(6)$. If $\psi = (f(z,w), g(z,w))$ is an automorphism (of the hypersurface) which preserves the origin and satisfies conditions similar to those in (20), then ψ is defined uniquely by these conditions, and thus can be denoted by $\psi_{\lambda ar}$. The mapping $\psi_{\lambda ar} \to \chi_{\lambda ar}$ defines a faithful representation of the stabilizer of a point (in the automorphism group of the hypersurface) in the stabilizer of a point of the sphere.

Suppose now that Γ is a germ, of a uniformly 2-nondegenerate hypersurface in C^3, given by an equation reduced to normal form up to the weight 9 inclusive. Let

$$\phi_{\lambda ar} = (f(z,\zeta,w), h(z,\zeta,w), g(z,\zeta,w))$$

be the (uniquely defined) automorphism preserving the origin and satisfying the conditions

$$\lambda = f'_z(0,0,0), \qquad a = \frac{f'_w(0,0,0)}{f'_z(0,0,0)}, \qquad r = \frac{\Re g''_{ww}(0,0,0)}{|f'_z(0,0,0)|^2}. \tag{21}$$

Then the following theorem holds.

Theorem 5. (*a*) *The mapping $\phi_{\lambda ar} \to \chi_{\lambda ar}$ is a faithful representation of the stabilizer of the origin of a uniformly* 2*-nondegenerate hypersurface of the form* (16) (*in C^3*) *in the stabilizer of a point of the sphere in C^2.*

(*b*) *If the hypersurface is a light cone, which can be represented in the form* [8]

$$\Im w = \frac{|z|^2 + \Re(z^2\bar{\zeta})}{1 - |\zeta|^2},$$

then the mapping given above defines an isomorphism of the stabilizer of a point of the cone and the stabilizer of a point of the sphere.

Recall that the automorphism group of the light cone is described in detail in [8].

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