

**Appendix to the paper  
“Classifications of  
homogeneous CR-manifolds  
in dimension 4”  
[arXiv:0911.1167](https://arxiv.org/abs/0911.1167)**

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**Type Ia for non-zero gamma.**

*In what follows we put  $n:=\text{gamma}$ ,  $n:=\text{delta}$ ,  $n:=\text{beta}$  depending on the  
case*

First step.

```
> restart:  
readlib(mtaylor):
```

```

A:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(y)-1:
T:=x*exp(y)+n*y*exp(y):
S1:=mtaylor(S,[z,cz],7): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=4*S2:
T3:=collect(16*(T2-n/2*S3),[z,cz],distributed):

```

Result check.

```

> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));

```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := -In + 1$$

$$T_{3,0,0,0} := 0$$

Introduce some notations.

```
> k:=1-I*n:ck:=1+I*n: m:=n^2+1:
```

Introduce new variables

```
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):
```

Write a mapping

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:
```

```
cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:
```

```
g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
```

```
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

Write the coefficients of the (3,4)-jet.

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
```

```
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
```

```
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
```

```
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
```

```
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
```

```
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
```

```
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
```

```
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
```

```
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
```

```
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
```

```
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
```

```

Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

Write the equations.

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

```

```

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

17 equations. Solve the system.

```

> s:=solve(EQ)[2];

```

$$s := \{h21 = -\frac{1}{6}r^4 n^4 - \frac{1}{3}r^4 n^2 - \frac{1}{6}r^4, F2 = -\frac{1}{6}r^2 n + f01,$$

$$H21 = \frac{1}{6}r^4 n^5 + 2r^2 n^4 f01 + \frac{1}{3}r^4 n^3 + 4r^2 f01 n^2 + \frac{1}{6}r^4 n + 2r^2 f01, g3 = 0, G3 = 0,$$

$$g001 = 0, H4 = 0, h4 = 0, h101 = 0, H101 = 0, G001 = -r n^2 f01 - r f01,$$

$$H02 = -\frac{1}{6}r^4 n^4 - \frac{1}{3}r^4 n^2 - \frac{1}{6}r^4, g11 = -\frac{1}{6}r^3 n^2 - \frac{1}{6}r^3,$$

$$G11 = \frac{1}{6}r^3 n^3 + 2r n^2 f01 + \frac{1}{6}r^3 n + 2r f01, r = r,$$

$$h02 = 2r^2 f01 + 4r^2 f01 n^2 + 2r^2 n^4 f01, F01 = -\frac{1}{12}r^2 n^2 - f01 n - \frac{1}{12}r^2, n = n,$$

$$f01 = f01, f2 = -\frac{1}{12}r^2 n^2 + \frac{1}{12}r^2 + f01 n\}$$

## Second Step.

```

> restart:
readlib(mtaylor):
A:=0:B:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(y)-1:

```

```

T:=x*exp(y)+n*y*exp(y):
S1:=mtaylor(S,[z,cz],7): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=4*S2:
T3:=collect(16*(T2-n/2*S3),[z,cz],distributed):
k:=1-I*n:ck:=1+I*n: m:=n^2+1:
Fix the results of the previous - (3,4) - step.
> f01:=a:
h21:= -1/6*r^4*n^4-1/3*r^4*n^2-1/6*r^4: G11:=
1/6*r^3*n^3+2*r*n^2*f01+1/6*r^3*n+2*r*f01: F01:= -1/12*r^2*n^2-
f01*n-1/12*r^2: G3:= 0: g3:= 0: H4:= 0: h4:= 0: g001:= 0: H101:= 0:
h101:= 0: f2:= -1/12*r^2*n^2+1/12*r^2+f01*n: h02:=
4*r^2*n^2*f01+2*r^2*n^4*f01+2*r^2*f01+r^3*A*n^4+2*r^3*A*n^2+r^3*A:
H21:=
1/3*r^4*n^3+4*r^2*n^2*f01+1/6*r^4*n+1/6*r^4*n^5+2*r^2*n^4*f01+2*r^2
*f01: G001:= -r*n^2*f01-r*f01: g11:= -1/6*r^3*n^2-1/6*r^3: H02:=
-1/6*r^4*n^4-1/3*r^4*n^2-1/6*r^4: F2:= -1/6*r^2*n+f01:

```

Introduce new variables

```

> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):

```

Write a mapping

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

```

```

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

```

```

cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

```

```

h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

```

```

ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

Write the relations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
```

```
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

Write the coefficients of the (4,5)-jet.

```

> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):

```

```

P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):

```

```

Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz)))
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-

```

```

Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

Write the equations and the variables.

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

```

```

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

Number of equations - 26.

```

> sol:=solve(EQ)[4];

```

$$\begin{aligned}
sol := \{ & H201 = -\frac{1}{72}r^5n^4 - \frac{1}{3}r^3n^3a + \frac{1}{72}r^5n^2 - 2ra^2n^2 - \frac{1}{6}r^3an + \frac{1}{72}r^5 - ra^2 \\
& - \frac{1}{72}n^6r^5 - \frac{1}{6}n^5r^3a - rn^4a^2, G02 = \frac{1}{6}r^2n^2a + \frac{1}{6}r^2a + \frac{1}{54}r^4n + \frac{1}{54}r^4n^3,
\end{aligned}$$

$$h31 = -\frac{1}{9}r^5n^3 - \frac{1}{18}r^5n^5 - \frac{1}{18}r^5n - \frac{2}{3}r^3a - \frac{2}{3}r^3an^4 - \frac{4}{3}r^3an^2,$$

$$G101 = -\frac{1}{72}r^4n^4 - \frac{1}{6}r^2n^3a - a^2n^2 - \frac{1}{6}r^2an + \frac{1}{72}r^4 - a^2,$$

$$g101 = \frac{1}{36} r^4 n^3 + \frac{1}{6} r^2 n^2 a + \frac{1}{36} r^4 n + \frac{1}{6} r^2 a,$$

$$H12 = -\frac{1}{18} r^5 n^3 - \frac{1}{36} r^5 n^5 - \frac{1}{36} r^5 n - \frac{1}{3} r^3 a - \frac{1}{3} r^3 a n^4 - \frac{2}{3} r^3 a n^2, h12 = \frac{1}{3} r^3 a n$$

$$+ \frac{1}{72} n^6 r^5 + \frac{1}{3} n^5 r^3 a + 2 r a^2 + 4 r a^2 n^2 + \frac{1}{24} r^5 + \frac{7}{72} r^5 n^2 + 2 r n^4 a^2 + \frac{5}{72} r^5 n^4$$

$$+ \frac{2}{3} r^3 n^3 a, H31 = \frac{4}{3} r^3 n^3 a + \frac{2}{3} n^5 r^3 a + \frac{2}{3} r^3 a n + \frac{1}{36} n^6 r^5 - \frac{1}{36} r^5 n^2 + 4 r a^2 - \frac{1}{36} r^5$$

$$+ 8 r a^2 n^2 + 4 r n^4 a^2 + \frac{1}{36} r^5 n^4, n = n, r = r, a = a,$$

$$h201 = \frac{1}{18} r^5 n^3 + \frac{1}{3} r^3 a n^2 + \frac{1}{36} r^5 n + \frac{1}{6} r^3 a + \frac{1}{36} r^5 n^5 + \frac{1}{6} r^3 a n^4, G4 = 0, H5 = 0,$$

$$h5 = 0, h011 = 0, g4 = 0, G21 = \frac{1}{36} r^4 n^4 + \frac{1}{2} r^2 a n + \frac{1}{2} r^2 n^3 a - \frac{1}{36} r^4 + 2 a^2 n^2 + 2 a^2,$$

$$H011 = \frac{1}{9} r^5 n^3 + \frac{2}{3} r^3 a n^2 + \frac{1}{18} r^5 n + \frac{1}{3} r^3 a + \frac{1}{18} r^5 n^5 + \frac{1}{3} r^3 a n^4,$$

$$g21 = -\frac{1}{18} r^4 n^3 - \frac{1}{2} r^2 n^2 a - \frac{1}{18} r^4 n - \frac{1}{2} r^2 a,$$

$$g02 = \frac{1}{6} r^2 a n + \frac{1}{6} r^2 n^3 a - 4 a^2 n^2 + \frac{7}{72} r^4 n^2 + \frac{1}{48} r^4 n^4 + \frac{11}{144} r^4 - 4 a^2,$$

$$F11 = \frac{1}{54} r^3 n - \frac{1}{3} r a, f3 = \frac{1}{72} \frac{r^4 n^3 - 3 r^4 n - 12 r^2 n^2 a + 72 n a^2 + 12 r^2 a}{r},$$

$$C1 = \frac{1}{36} \frac{-r^4 n^2 - 3 r^4 + 216 a^2}{r^2}, F3 = \frac{1}{72} \frac{3 r^4 n^2 - 24 r^2 a n - r^4 + 72 a^2}{r},$$

$$F001 = \frac{1}{144} \frac{r^4 n^3 + 12 r^2 n^2 a + r^4 n + 72 n a^2 + 12 r^2 a}{r}, C2 = \frac{1}{54} r^2 n,$$

$$f11 = -\frac{1}{54} n r (-r^2 n + 18 a), f001 = -\frac{1}{144} \frac{-r^4 n^2 - r^4 + 72 a^2}{r} \}$$

> CC1 := -1/36\*(r^4\*n^2+3\*r^4-216\*a^2)/(r^2);  
 CC2 := 1/54\*n\*r^2;



$$CCI := -\frac{1}{36} \frac{r^4 n^2 + 3 r^4 - 216 a^2}{r^2}$$

$$CC2 := \frac{1}{54} r^2 n$$

Thus if gamma is non-zero, then it is not the cubic.

Type Ia for gamma=0.

>

First step: (3,4)-normalization -----

(2,3,4) jet

> restart:

readlib(mtaylor):

A:=0:

x:=(z+cz)/2:y:=(z-cz)/(2\*I):

S:=exp(y)-1:

T:=x\*exp(y):

S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):

S2:=int(diff(int(diff(S1,z),z),cz),cz):

T2:=int(diff(int(diff(T1,z),z),cz),cz):

S3:=4\*S2:

T3:=16\*T2:

k:=1:ck:=1: m:=1:

**Result check.**

> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z))));

S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz))));

S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z))));

print('xxxxxxxxxxxxxxxxxxxxxxxxxxxx');

T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z))));

T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz))));

T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z))));

T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))/2);

T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z))));

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 1$$

$$T_{3,0,0,0} := 0$$

### Introduce new variables

```
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):
```

### Write a mapping

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

### Write the relations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

### Write the coefficients of the (3,4)-jet.

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
```

```

P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):

P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):

Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write the equations.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**17 equations      Solve the system.**

```

> s:=solve(EQ)[2];

```

$$s := \{H101 = 0, h101 = 0, h4 = 0, g001 = 0, G3 = 0, g3 = 0, H4 = 0, f2 = \frac{1}{12} r^2, F2 = f01, \\ G11 = 2 r f01, g11 = -\frac{1}{6} r^3, G001 = -r f01, h21 = -\frac{1}{6} r^4, H02 = -\frac{1}{6} r^4, F01 = -\frac{1}{12} r^2, \\ h02 = 2 r^2 f01, r = r, f01 = f01, H21 = 2 r^2 f01\}$$

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```
> restart:
readlib(mtaylor):
A:=0: B:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(y)-1:
T:=x*exp(y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=4*S2:
T3:=16*T2:
k:=1:ck:=1: m:=1:

f01:=a:
H101:= 0: h101:= 0: h4:= 0: g001:= 0: G3:= 0: g3:= 0: H4:= 0: f2:=
1/12*r^2: F2:= f01: G11:= 2*r*f01: g11:= -1/6*r^3: G001:= -r*f01:
h21:= -1/6*r^4: H02:= -1/6*r^4: F01:= -1/12*r^2: h02:= 2*r^2*f01:
H21:= 2*r^2*f01:

Introduce new variables
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):

Write a mapping
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

g:=
```

```

r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

```

```

cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

```

```

h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

```

```

ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

**Write the relations for the mapping.**

```

> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):

```

**Write the coefficients of the (4,5)-jet.**

```

> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):

```

```

P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):

```

```

Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):

```

```

Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz)))
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

```

```

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Number of equations - 26.**

```

> sol:=solve(EQ)[2];

```

$$sol := \{f001 = -\frac{1}{144} \frac{-r^4 + 72 a^2}{r}, g02 = \frac{11}{144} r^4 - 4 a^2, H12 = -\frac{1}{3} r^3 a, h31 = -\frac{2}{3} r^3 a, a = a,$$

$$g4 = 0, h5 = 0, G4 = 0, h011 = 0, H5 = 0, r = r, h201 = \frac{1}{6} r^3 a,$$

$$H201 = -\frac{1}{72}r(-r^4 + 72a^2), g21 = -\frac{1}{2}r^2 a, C1 = \frac{1}{12}\frac{-r^4 + 72a^2}{r^2}, G101 = \frac{1}{72}r^4 - a^2,$$

$$g101 = \frac{1}{6}r^2 a, f11 = 0, G21 = -\frac{1}{36}r^4 + 2a^2, F3 = \frac{1}{72}\frac{-r^4 + 72a^2}{r}, F001 = \frac{1}{12}ra,$$

$$C2 = 0, F11 = -\frac{1}{3}ra, H011 = \frac{1}{3}r^3 a, f3 = \frac{1}{6}ra, G02 = \frac{1}{6}r^2 a, H31 = -\frac{1}{36}r^5 + 4ra^2,$$

$$h12 = \frac{1}{24}r^5 + 2ra^2\}$$

> CC1:=1/12\*(-r^4+72\*a^2)/(r^2);

$$CC1 := \frac{1}{12}\frac{-r^4 + 72a^2}{r^2}$$

> solve(CC1=0, a);

$$\frac{1}{12}\sqrt{2}r^2, -\frac{1}{12}\sqrt{2}r^2$$

### Third Step: (5,6)-normalization ----- (4,5,6) jet

> restart:

readlib(mtaylor):

A:=0: B:=0:C1:=0:C2:=0:a:=p\*r^2:

x:=(z+cz)/2:y:=(z-cz)/(2\*I):

S:=exp(y)-1:

T:=x\*exp(y):

S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):

S2:=int(diff(int(diff(S1,z),z),cz),cz):

T2:=int(diff(int(diff(T1,z),z),cz),cz):

S3:=4\*S2:

T3:=16\*T2:

k:=1:ck:=1: m:=1:

f01:=a:

H101:= 0: h101:= 0: h4:= 0: g001:= 0: G3:= 0: g3:= 0: H4:= 0: f2:=

1/12\*r^2: F2:= f01: G11:= 2\*r\*f01: g11:= -1/6\*r^3: G001:= -r\*f01:

h21:= -1/6\*r^4: H02:= -1/6\*r^4: F01:= -1/12\*r^2: h02:= 2\*r^2\*f01:

H21:= 2\*r^2\*f01:

f001:= -1/144\*(-r^4+72\*a^2)/r: g02:= 11/144\*r^4-4\*a^2: H12:=

-1/3\*r^3\*a: h31:= -2/3\*r^3\*a: g4:= 0: h5:= 0: G4:= 0:

h011:= 0: H5:= 0: h201:= 1/6\*r^3\*a: H201:= -1/72\*r\*(-r^4+72\*a^2):

g21:= -1/2\*r^2\*a: G101:= 1/72\*r^4-a^2: g101:= 1/6\*r^2\*a: f11:= 0:

```
G21:= -1/36*r^4+2*a^2: F3:= 1/72*(-r^4+72*a^2)/r: F001:= 1/12*r*a:
F11:= -1/3*r*a: H011:= 1/3*r^3*a: f3:= 1/6*r*a: G02:= 1/6*r^2*a:
H31:= -1/36*r^5+4*r*a^2: h12:= 1/24*r^5+2*r*a^2:
```

**Write a mapping**

```
> f:=
```

```
r*ck*z+
(a+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3+
(F4+I*f4)*z^4+(F21+I*f21)*z^2*w2+(F02+I*f02)*w2^2+
(F101+I*f101)*z*w3:
```

```
cf:=
```

```
r*k*cz+
(a-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3+
(F4-I*f4)*cz^4+(F21-I*f21)*cz^2*cw2+(F02-I*f02)*cw2^2+(F101-
I*f101)*cz*cw3:
```

```
g:=
```

```
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3+
(G5+I*g5)*z^5+(G31+I*g31)*z^3*w2+(G12+I*g12)*z*w2^2+
(G201+I*g201)*z^2*w3+(G011+I*g011)*w2*w3:
```

```
cg:=
```

```
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3+
(G5-I*g5)*cz^5+(G31-I*g31)*cz^3*cw2+(G12-I*g12)*cz*cw2^2+(G201-
I*g201)*cz^2*cw3+(G011-I*g011)*cw2*cw3:
```

```
h:=
```

```
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3+
(H6+I*h6)*z^6+(H41+I*h41)*z^4*w2+(H22+I*h22)*z^2*w2^2+
(H03+I*h03)*w2^3+(H301+I*h301)*z^3*w3+(H111+I*h111)*z*w2*m^2*w3+
(H002+I*h002)*w3^2:
```

```
ch:=
```

```
r^3*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
```



```
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cz^2+(H12-I*h12)*cz*cz^2^2+(H201-I*h201)*cz^2*cz^3+(H011-I*h011)*cz^2*cz^3+
(H6-I*h6)*cz^6+(H41-I*h41)*cz^4*cz^2+(H22-I*h22)*cz^2*cz^2^2+(H03-I*h03)*cz^2^3+(H301-I*h301)*cz^3*cz^3+(H111-I*h111)*cz*cz^2*cz^3+(H002-I*h002)*cz^3^2:
```

### Introduce new variables

```
> w2:=u2+I*(z*cz+B*z^2*cz^2+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+
(E1-I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
cw2:=u2-I*(z*cz+B*z^2*cz^2+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+(E1-I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
w3:=u3+I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2+J*z^3*cz^3):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2+J*z^3*cz^3):
```

### Write the relations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

### Write the coefficients of the (5,6)-jet.

```
>
P[5,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,z)))):
P[4,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,cz)))):
P[3,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,cz,cz)))):
P[3,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,u2)))):
P[2,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz,u2)))):
P[1,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2,u2)))):
P[2,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u3)))):
P[1,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u3)))):
P[0,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u2,u3)))):
```

```
P1[5,0,0,0]:=subs(I=0,P[5,0,0,0]):
P2[5,0,0,0]:=normal(expand((P[5,0,0,0]-P1[5,0,0,0])*I)):
P1[4,1,0,0]:=subs(I=0,P[4,1,0,0]):
P2[4,1,0,0]:=normal(expand((P[4,1,0,0]-P1[4,1,0,0])*I)):
P1[3,2,0,0]:=subs(I=0,P[3,2,0,0]):
P2[3,2,0,0]:=normal(expand((P[3,2,0,0]-P1[3,2,0,0])*I)):
P1[3,0,1,0]:=subs(I=0,P[3,0,1,0]):
P2[3,0,1,0]:=normal(expand((P[3,0,1,0]-P1[3,0,1,0])*I)):
P1[2,1,1,0]:=subs(I=0,P[2,1,1,0]):
```

```

P2[2,1,1,0]:=normal(expand((P[2,1,1,0]-P1[2,1,1,0])*I)):
P1[1,0,2,0]:=subs(I=0,P[1,0,2,0]):
P2[1,0,2,0]:=normal(expand((P[1,0,2,0]-P1[1,0,2,0])*I)):
P1[2,0,0,1]:=subs(I=0,P[2,0,0,1]):
P2[2,0,0,1]:=normal(expand((P[2,0,0,1]-P1[2,0,0,1])*I)):

Q[6,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z)))):
Q[5,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z,cz)))):
Q[4,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,cz,cz)))):
Q[3,3,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz,cz,cz)))):
Q[4,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,u2)))):
Q[3,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz,u2)))):
Q[2,2,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz,u2)))):
Q[2,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2,u2)))):
Q[1,1,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2,u2)))):
Q[0,0,3,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u2,u2)))):
Q[3,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u3)))):
Q[2,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u3)))):
Q[1,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u3)))):
Q[0,0,0,2]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u3,u3)))):

```

```

Q1[6,0,0,0]:=subs(I=0,Q[6,0,0,0]):
Q2[6,0,0,0]:=normal(expand((Q[6,0,0,0]-Q1[6,0,0,0])*I)):
Q1[5,1,0,0]:=subs(I=0,Q[5,1,0,0]):
Q2[5,1,0,0]:=normal(expand((Q[5,1,0,0]-Q1[5,1,0,0])*I)):
Q1[4,2,0,0]:=subs(I=0,Q[4,2,0,0]):
Q2[4,2,0,0]:=normal(expand((Q[4,2,0,0]-Q1[4,2,0,0])*I)):

Q1[4,0,1,0]:=subs(I=0,Q[4,0,1,0]):
Q2[4,0,1,0]:=normal(expand((Q[4,0,1,0]-Q1[4,0,1,0])*I)):
Q1[3,1,1,0]:=subs(I=0,Q[3,1,1,0]):
Q2[3,1,1,0]:=normal(expand((Q[3,1,1,0]-Q1[3,1,1,0])*I)):

```

```

Q1[2,0,2,0]:=subs(I=0,Q[2,0,2,0]):
Q2[2,0,2,0]:=normal(expand((Q[2,0,2,0]-Q1[2,0,2,0])*I)):

Q1[3,0,0,1]:=subs(I=0,Q[3,0,0,1]):
Q2[3,0,0,1]:=normal(expand((Q[3,0,0,1]-Q1[3,0,0,1])*I)):
Q1[2,1,0,1]:=subs(I=0,Q[2,1,0,1]):
Q2[2,1,0,1]:=normal(expand((Q[2,1,0,1]-Q1[2,1,0,1])*I)):
Q1[1,0,1,1]:=subs(I=0,Q[1,0,1,1]):
Q2[1,0,1,1]:=normal(expand((Q[1,0,1,1]-Q1[1,0,1,1])*I)):

```

### Write the equations -39.

```

> EQ:={
P1[5,0,0,0]=0, P2[5,0,0,0]=0,
P1[4,1,0,0]=0, P2[4,1,0,0]=0,
P1[3,2,0,0]=0, P2[3,2,0,0]=0,
P1[3,0,1,0]=0, P2[3,0,1,0]=0,
P1[2,1,1,0]=0, P2[2,1,1,0]=0,
P1[1,0,2,0]=0, P2[1,0,2,0]=0,
P1[2,0,0,1]=0, P2[2,0,0,1]=0,
P[1,1,0,1]=0,
P[0,0,1,1]=0,

Q1[6,0,0,0]=0,Q2[6,0,0,0]=0,
Q1[5,1,0,0]=0,Q2[5,1,0,0]=0,
Q1[4,2,0,0]=0,Q2[4,2,0,0]=0,
Q[3,3,0,0]=0,
Q1[4,0,1,0]=0,Q2[4,0,1,0]=0,
Q1[3,1,1,0]=0,Q2[3,1,1,0]=0,
Q[2,2,1,0]=0,
Q1[2,0,2,0]=0,Q2[2,0,2,0]=0,
Q[1,1,2,0]=0,
Q[0,0,3,0]=0,
Q1[3,0,0,1]=0,Q2[3,0,0,1]=0,
Q1[2,1,0,1]=0,Q2[2,1,0,1]=0,
Q1[1,0,1,1]=0,Q2[1,0,1,1]=0,
Q[0,0,0,2]=0
}:
> solve(EQ);
{H002 = 0, h301 = 0, F02 = F02, f02 = f02, H301 = 0, g011 = 0, G201 = 0, g201 = 0, J = J,
  F101 = F101, g12 = 0, F4 = F4, f4 = f4, F21 = F21, f21 = f21, h41 = 0, H41 = 0,
  G31 = 0, r = 0, f101 = f101, E2 = E2, E1 = E1, D2 = D2, D1 = D1, G5 = 0, g5 = 0,
  h6 = 0, H6 = 0, h002 = 0, H111 = 0, g31 = 0, h03 = 0, h111 = 0, G011 = 0, h22 = 0,
  p = p, H22 = 0, G12 = 0, H03 = 0}

```

>

There is no solution with non-zero  $r$  and we see, that for  $\gamma=0$  it is not a cubic as well.

## Type Ib for $\gamma$ - non-zero.

First step: (3,4)-normalization ----- (2,3,4)  
jet

```

> restart:
A:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
Original equations v_2=1/(1+y)-1, v_3=x/(1+y)+gamma*ln(1+y), .
The origin is the center of the germ
> S:=1/(1+y)-1:
T:=x/(1+y)+n*ln(1+y):
Load a package.
> readlib(mtaylor):
Tae the Theilor expansion of F and G, kill the harmonic terms and bring to the standard form
v2=|z|^2+O(3), v3=2Re k z^2 cz +O(4).
> S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=2*S2:
T3:=collect(16*(T2+n/4*S3),[z,cz],distributed):
Result check.
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));

```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 2 + In$$

$$T_{3,0,0,0} := 0$$

> k:=2+I\*n:ck:=2-I\*n: m:=n^2+4:

### Introduce new variables

> w2:=u2+I\*z\*cz: cw2:=u2-I\*z\*cz:

w3:=u3+I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):

cw3:=u3-I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):

### Write a mapping

> f:=

r\*ck\*z+

(F2+I\*f2)\*z^2+(F01+I\*f01)\*w2:

cf:=

r\*k\*cz+

(F2-I\*f2)\*cz^2+(F01-I\*f01)\*cw2:

g:=

r^2\*m\*w2+

(G3+I\*g3)\*z^3+(G11+I\*g11)\*z\*w2+(G001+I\*g001)\*w3:

cg:=

r^2\*m\*cw2+

(G3-I\*g3)\*cz^3+(G11-I\*g11)\*cz\*cw2+(G001-I\*g001)\*cw3:

h:=

r^3\*m^2\*w3+

(H4+I\*h4)\*z^4+(H21+I\*h21)\*z^2\*w2+(H02+I\*h02)\*w2^2+

(H101+I\*h101)\*z\*w3:

ch:=

r^3\*m^2\*cw3+

(H4-I\*h4)\*cz^4+(H21-I\*h21)\*cz^2\*cw2+(H02-I\*h02)\*cw2^2+(H101-

I\*h101)\*cz\*cw3:

### Write the relations for the mapping.

> e2:=subs([z=f,cz=cf],2\*S3)+I\*(g-cg):

e3:=subs([z=f,cz=cf],2\*T3)+I\*(h-ch):

### Write the coefficients of the (3,4)-jet.

> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):

```

P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):

P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):

Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write 17 equations.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

```

```

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

>

**Solve the system.**

```

> solve(EQ)[2];

```

$$\{h02 = 8 r^2 f01 n^2 + r^2 n^4 f01 + 16 r^2 f01, n = n, r = r, F01 = \frac{1}{4} r^2 n^2 + \frac{1}{2} f01 n + r^2,$$

$$h21 = 8 r^4 n^2 + 16 r^4 + r^4 n^4, F2 = -r^2 n + f01, f01 = f01,$$

$$H21 = 4 r^4 n^3 + 8 r^2 f01 n^2 + 8 r^4 n + 16 r^2 f01 + \frac{1}{2} r^4 n^5 + r^2 n^4 f01,$$

$$G001 = -\frac{1}{2} r n^2 f01 - 2 r f01, H02 = 8 r^4 n^2 + 16 r^4 + r^4 n^4, g11 = r^3 n^2 + 4 r^3,$$

$$f2 = \frac{1}{4} r^2 n^2 - r^2 - \frac{1}{2} f01 n, g001 = 0, H101 = 0, h101 = 0, h4 = 0, H4 = 0, g3 = 0,$$

$$G3 = 0, G11 = \frac{1}{2} r^3 n^3 + r n^2 f01 + 2 r^3 n + 4 r f01 \}$$

>  
>

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```
> restart: readlib(mttaylor):A:=0:B:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=1/(1+y)-1:
T:=x/(1+y)+n*ln(1+y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=2*S2:
T3:=collect(16*(T2+n/4*S3),[z,cz],distributed):
k:=2+I*n:ck:=2-I*n: m:=n^2+4:
```

**Write the results of the previous step.**

```
> f01:= a: h4:= 0: g001:= 0: H101:= 0: h101:= 0: H4:= 0: g3:= 0:
G3:= 0: H21:=
4*r^4*n^3+8*r^2*f01*n^2+8*r^4*n+16*r^2*f01+1/2*r^4*n^5+r^2*n^4*f01:
G001:= -1/2*r*n^2*f01-2*r*f01: F01:= 1/4*r^2*n^2+1/2*f01*n+r^2:
F2:= -r^2*n+f01: g11:= r^3*n^2+4*r^3: H02:=
8*r^4*n^2+16*r^4+r^4*n^4: f2:= 1/4*r^2*n^2-r^2-1/2*f01*n: G11:=
1/2*r^3*n^3+r*n^2*f01+2*r^3*n+4*r*f01: h21:=
8*r^4*n^2+16*r^4+r^4*n^4: h02:=
8*r^2*f01*n^2+r^2*n^4*f01+16*r^2*f01:
```

**Introduce new variables**

```
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
```

**Write a mapping**

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

**Write the relations for the mapping.**

```

> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):

```

**Write the coefficients of the (4,5)-jet.**

```

> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):

```



```

P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):

```

```

Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

```

```

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,

```

$Q1[1, 0, 2, 0]=0, Q2[1, 0, 2, 0]=0,$   
 $Q1[2, 0, 0, 1]=0, Q2[2, 0, 0, 1]=0,$   
 $Q[1, 1, 0, 1]=0,$   
 $Q[0, 0, 1, 1]=0$   
 $\}:$

**Number of equations - 26.**

> solve(EQ) [4];

$$\{h31 = 16 r^3 a n^2 + 32 r^3 a + 2 r^3 a n^4 + 8 r^5 n^3 + r^5 n^5 + 16 r^5 n, a = a, n = n,$$

$$G02 = -\frac{2}{3} r^4 n - 2 r^2 a - \frac{1}{6} r^4 n^3 - \frac{1}{2} r^2 a n^2, r = r,$$

$$H011 = -4 r^5 n^3 - 8 r^3 a n^2 - 8 r^5 n - 16 r^3 a - \frac{1}{2} r^5 n^5 - r^3 a n^4,$$

$$H12 = 8 r^3 a n^2 + 16 r^3 a + r^3 a n^4 + 4 r^5 n^3 + \frac{1}{2} r^5 n^5 + 8 r^5 n, H31 = n^5 r^3 a + 16 r^3 a n$$

$$+ 8 r^3 a n^3 + \frac{1}{4} n^6 r^5 + r n^4 a^2 + 16 r a^2 - 16 r^5 + r^5 n^4 - 4 r^5 n^2 + 8 r a^2 n^2,$$

$$G21 = \frac{1}{4} r^4 n^4 + \frac{1}{2} a^2 n^2 - 4 r^4 + 2 a^2 + \frac{3}{4} r^2 a n^3 + 3 r^2 a n, h011 = 0, h5 = 0, H5 = 0,$$

$$G4 = 0, g4 = 0, h12 = \frac{1}{2} r n^4 a^2 + 8 r a^2 + 8 r^3 a n + 24 r^5 + 14 r^5 n^2 + 4 r^3 a n^3$$

$$+ \frac{5}{2} r^5 n^4 + 4 r a^2 n^2 + \frac{1}{2} n^5 r^3 a + \frac{1}{8} n^6 r^5, g101 = -\frac{1}{4} r^4 n^3 - \frac{1}{2} r^2 a n^2 - r^4 n - 2 r^2 a,$$

$$g21 = r^4 n^3 + 4 r^4 n + \frac{3}{2} r^2 a n^2 + 6 r^2 a,$$

$$G101 = -\frac{1}{16} r^4 n^4 - \frac{1}{4} r^2 a n^3 - \frac{1}{4} a^2 n^2 - r^2 a n - a^2 + r^4, F11 = -\frac{1}{3} r^3 n + 2 r a,$$

$$C1 = -\frac{1}{8} \frac{r^4 n^2 - 12 a^2 + 12 r^4}{r^2}, F3 = \frac{1}{8} \frac{3 r^4 n^2 - 8 r^2 a n + 4 a^2 - 4 r^4}{r},$$

$$F001 = -\frac{1}{32} \frac{r^4 n^3 + 4 r^2 a n^2 + 4 n a^2 + 4 r^4 n + 16 r^2 a}{r}, C2 = -\frac{1}{6} r^2 n,$$

$$f11 = \frac{1}{6} n r (r^2 n - 6 a), f3 = -\frac{1}{16} \frac{r^4 n^3 - 12 r^4 n - 4 r^2 a n^2 + 4 n a^2 + 16 r^2 a}{r},$$

$$f001 = \frac{1}{16} \frac{r^4 n^2 - 4 a^2 + 4 r^4}{r},$$

$$g02 = -a^2 n^2 + \frac{1}{8} r^4 n^4 + \frac{1}{4} r^2 a n^3 + 2 r^4 n^2 + r^2 a n - 4 a^2 + 6 r^4,$$

$$\begin{aligned} h201 = & -\frac{1}{4} r^5 n^5 - \frac{1}{2} r^3 a n^4 - 2 r^5 n^3 - 4 r^3 a n^2 - 4 r^5 n - 8 r^3 a, H201 = -\frac{1}{4} r^5 n^4 \\ & - 2 r^3 a n^3 - 2 r a^2 n^2 + r^5 n^2 - 4 r^3 a n - 4 r a^2 + 4 r^5 - \frac{1}{16} n^6 r^5 - \frac{1}{4} n^5 r^3 a - \frac{1}{4} r n^4 a^2 \\ & \} \end{aligned}$$

> solve(EQ) [4];

$$\{h31 = 16 r^3 a n^2 + 32 r^3 a + 2 r^3 a n^4 + 8 r^5 n^3 + r^5 n^5 + 16 r^5 n, a = a, n = n,$$

$$G02 = -\frac{2}{3} r^4 n - 2 r^2 a - \frac{1}{6} r^4 n^3 - \frac{1}{2} r^2 a n^2, r = r,$$

$$H011 = -4 r^5 n^3 - 8 r^3 a n^2 - 8 r^5 n - 16 r^3 a - \frac{1}{2} r^5 n^5 - r^3 a n^4,$$

$$\begin{aligned} H12 = & 8 r^3 a n^2 + 16 r^3 a + r^3 a n^4 + 4 r^5 n^3 + \frac{1}{2} r^5 n^5 + 8 r^5 n, H31 = n^5 r^3 a + 16 r^3 a n \\ & + 8 r^3 a n^3 + \frac{1}{4} n^6 r^5 + r n^4 a^2 + 16 r a^2 - 16 r^5 + r^5 n^4 - 4 r^5 n^2 + 8 r a^2 n^2, \end{aligned}$$

$$G21 = \frac{1}{4} r^4 n^4 + \frac{1}{2} a^2 n^2 - 4 r^4 + 2 a^2 + \frac{3}{4} r^2 a n^3 + 3 r^2 a n, h011 = 0, h5 = 0, H5 = 0,$$

$$G4 = 0, g4 = 0, h12 = \frac{1}{2} r n^4 a^2 + 8 r a^2 + 8 r^3 a n + 24 r^5 + 14 r^5 n^2 + 4 r^3 a n^3$$

$$+ \frac{5}{2} r^5 n^4 + 4 r a^2 n^2 + \frac{1}{2} n^5 r^3 a + \frac{1}{8} n^6 r^5, g101 = -\frac{1}{4} r^4 n^3 - \frac{1}{2} r^2 a n^2 - r^4 n - 2 r^2 a,$$

$$g21 = r^4 n^3 + 4 r^4 n + \frac{3}{2} r^2 a n^2 + 6 r^2 a,$$

$$G101 = -\frac{1}{16} r^4 n^4 - \frac{1}{4} r^2 a n^3 - \frac{1}{4} a^2 n^2 - r^2 a n - a^2 + r^4, F11 = -\frac{1}{3} r^3 n + 2 r a,$$

$$CI = -\frac{1}{8} \frac{r^4 n^2 - 12 a^2 + 12 r^4}{r^2}, F3 = \frac{1}{8} \frac{3 r^4 n^2 - 8 r^2 a n + 4 a^2 - 4 r^4}{r},$$

$$F001 = -\frac{1}{32} \frac{r^4 n^3 + 4 r^2 a n^2 + 4 n a^2 + 4 r^4 n + 16 r^2 a}{r}, C2 = -\frac{1}{6} r^2 n,$$

$$f11 = \frac{1}{6} n r (r^2 n - 6 a), f3 = -\frac{1}{16} \frac{r^4 n^3 - 12 r^4 n - 4 r^2 a n^2 + 4 n a^2 + 16 r^2 a}{r},$$

$$f001 = \frac{1}{16} \frac{r^4 n^2 - 4 a^2 + 4 r^4}{r},$$

$$g02 = -a^2 n^2 + \frac{1}{8} r^4 n^4 + \frac{1}{4} r^2 a n^3 + 2 r^4 n^2 + r^2 a n - 4 a^2 + 6 r^4,$$

$$h201 = -\frac{1}{4} r^5 n^5 - \frac{1}{2} r^3 a n^4 - 2 r^5 n^3 - 4 r^3 a n^2 - 4 r^5 n - 8 r^3 a, H201 = -\frac{1}{4} r^5 n^4$$

$$- 2 r^3 a n^3 - 2 r a^2 n^2 + r^5 n^2 - 4 r^3 a n - 4 r a^2 + 4 r^5 - \frac{1}{16} n^6 r^5 - \frac{1}{4} n^5 r^3 a - \frac{1}{4} r n^4 a^2$$

$$\}$$

> CC1 := -1/8 \* (-12\*a^2+r^4\*n^2+12\*r^4) / (r^2);  
 CC2 := -1/6\*r^2\*n;

$$CCI := -\frac{1}{8} \frac{r^4 n^2 - 12 a^2 + 12 r^4}{r^2}$$

$$CC2 := -\frac{1}{6} r^2 n$$

So, if gamma is non-zero, then it is not the cubic.

>

Type Ib for gamma=0 .

First step: (3,4)-normalization -----

(2,3,4) jet

```
> restart:
readlib(mttaylor):
A:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=1/(1+y)-1:
T:=x/(1+y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
```

S3:=2\*S2:  
 T3:=8\*T2:

**Result check.**

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 1$$

$$T_{3,0,0,0} := 0$$

**Introduce the notations.**

```
> k:=1:ck:=1: m:=1:
```

**Introduce new variables**

```
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):
```

**Write a mapping**

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:
```

```
cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:
```

```
g:=
```

```
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Check that (2,3)-jet is zero.**

```
> P[0,0,0,0]:=subs([z=0,cz=0,u2=0,u3=0],e2);
P[1,0,0,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e2,z));
P[2,0,0,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z));
P[1,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz)));
P[0,0,1,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e2,u2));

Q[0,0,0,0]:=subs([z=0,cz=0,u2=0,u3=0],e3);
Q[1,0,0,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e3,z));
Q[2,0,0,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z));
Q[1,1,0,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz));
Q[0,0,1,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2));
Q[3,0,0,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z));
Q[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz)));
Q[1,0,1,0]:=subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2));
Q[0,0,0,1]:=subs([z=0,cz=0,u2=0,u3=0],diff(e3,u3));
```

$$P_{0,0,0,0} := 0$$

$$P_{1,0,0,0} := 0$$

$$P_{2,0,0,0} := 0$$

$$P_{1,1,0,0} := 0$$

$$P_{0,0,1,0} := 0$$

$$Q_{0,0,0,0} := 0$$

$$Q_{1,0,0,0} := 0$$

$$Q_{2,0,0,0} := 0$$

$$Q_{1,1,0,0} := 0$$

$$Q_{0,0,1,0} := 0$$

$$Q_{3,0,0,0} := 0$$

$$Q_{2,1,0,0} := 0$$

$$Q_{1,0,1,0} := 0$$

$$Q_{0,0,0,1} := 0$$

**Write the coefficients of the (3,4)-jet.**

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
```

Q1[1,0,0,1])\*I):

**Write 17 equations.**

```
> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,
```

```
Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**Solve the system.**

```
> solve(EQ)[2];
```

$\{f01 = f01, F2 = f01, h21 = \frac{1}{2}r^4, g001 = 0, H101 = 0, h101 = 0, G3 = 0, g3 = 0, H4 = 0,$

$h4 = 0, r = r, G11 = 2rf01, G001 = -rf01, g11 = \frac{1}{2}r^3, H21 = 2r^2f01, H02 = \frac{1}{2}r^4,$

$f2 = -\frac{1}{4}r^2, h02 = 2r^2f01, F01 = \frac{1}{4}r^2\}$

>

**Second Step: (4,5)-normalization -----**

**(3,4,5) jet**

```
> restart:
```

```
readlib(mtaylor):
```

```
A:=0:B:=0:
```

```
x:=(z+cz)/2:y:=(z-cz)/(2*I):
```

```
S:=1/(1+y)-1:
```

```
T:=x/(1+y):
```

```
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
```

```
S2:=int(diff(int(diff(S1,z),z),cz),cz):
```

```
T2:=int(diff(int(diff(T1,z),z),cz),cz):
```

```
S3:=2*S2:
```

```
T3:=8*T2:
```

```
k:=1:ck:=1: m:=1:
```

**Write the results of the previous step.**

```
> f01:= a:
```

```
F2:= f01: h21:= 1/2*r^4: g001:= 0: H101:= 0: h101:= 0: G3:= 0: g3:=
0: H4:= 0: h4:= 0: G11:= 2*r*f01: G001:= -r*f01: g11:= 1/2*r^3:
H21:= 2*r^2*f01: H02:= 1/2*r^4: f2:= -1/4*r^2: h02:= 2*r^2*f01:
```



```
F01:= 1/4*r^2:
```

### Introduce new variables

```
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
```

```
w3:=u3+I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-I
```

```
I*C2)*cz^3*z^2):  
cw3:=u3-I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-
```

### Write a mapping

```
> f:=
```

```
r*ck*z+
```

```
(F2+I*f2)*z^2+(F01+I*f01)*w2+
```

```
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:
```

```
cf:=
```

```
r*k*cz+
```

```
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
```

```
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:
```

```
g:=
```

```
r^2*m*w2+
```

```
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
```

```
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
```

```
(G101+I*g101)*z*w3:
```

```
cg:=
```

```
r^2*m*cw2+
```

```
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
```

```
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
```

```
I*g101)*cz*cw3:
```

```
h:=
```

```
r^3*m^2*w3+
```

```
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
```

```
(H101+I*h101)*z*w3+
```

```
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
```

```
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:
```

```
ch:=
```

```
r^3*m^2*cw3+
```

```
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
```

```
I*h101)*cz*cw3+
```

```
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
```

```
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:
```

### Write the relations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
```

$e3 := \text{subs}([z=f, cz=cf], 2*T3) + I*(h-ch) :$

**Check that (3,4)-jet is zero.**

```
> P[3,0,0,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z)));
P[2,1,0,0] := factor(expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, cz)
)));
P[1,0,1,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, u2)));
P[0,0,0,1] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, u3)));

Q[4,0,0,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, z)));
Q[3,1,0,0] := factor(expand(subs([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, cz)
)));
Q[2,2,0,0] := factor(expand(subs([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, cz,
cz))));
Q[2,0,1,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, u2)));
Q[1,1,1,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e3, z, cz, u2)));
Q[1,0,0,1] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e3, z, u3)));
```

$$P_{3,0,0,0} := 0$$

$$P_{2,1,0,0} := 0$$

$$P_{1,0,1,0} := 0$$

$$P_{0,0,0,1} := 0$$

$$Q_{4,0,0,0} := 0$$

$$Q_{3,1,0,0} := 0$$

$$Q_{2,2,0,0} := 0$$

$$Q_{2,0,1,0} := 0$$

$$Q_{1,1,1,0} := 0$$

$$Q_{1,0,0,1} := 0$$

**Write the coefficients of the (4,5)-jet.**

```
> P[4,0,0,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z, z)));
P[3,1,0,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z, cz)));
P[2,2,0,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, cz, cz)));
P[2,0,1,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, u2)));
P[1,1,1,0] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, cz, u2)));
P[1,0,0,1] := expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, u3)));
```

```

P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):

```

```

Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

```

```

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,

```

```

Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Number of equations - 26.**

```
> solve(EQ) [2];
```

$$\{G101 = -a^2 + \frac{1}{16}r^4, g101 = -\frac{1}{2}r^2 a, G4 = 0, g4 = 0, H5 = 0, h5 = 0, h011 = 0,$$

$$H201 = -\frac{1}{16}r(16a^2 - r^4), H011 = -r^3 a, G21 = -\frac{1}{4}r^4 + 2a^2, H12 = r^3 a,$$

$$h31 = 2r^3 a, g21 = \frac{3}{2}r^2 a, H31 = -\frac{1}{4}r^5 + 4ra^2, h201 = -\frac{1}{2}r^3 a, f11 = 0,$$

$$F001 = -\frac{1}{4}ra, C2 = 0, a = a, F11 = ra, r = r, f001 = -\frac{1}{32}\frac{16a^2 - r^4}{r},$$

$$F3 = \frac{1}{16}\frac{16a^2 - r^4}{r}, g02 = -4a^2 + \frac{3}{8}r^4, G02 = -\frac{1}{2}r^2 a, f3 = -\frac{1}{2}ra, C1 = \frac{3}{8}\frac{16a^2 - r^4}{r^2},$$

$$h12 = \frac{3}{8}r^5 + 2ra^2\}$$

```
> CC1:=3/8*(16*a^2-r^4)/(r^2);
```

$$CC1 := \frac{3}{8}\frac{16a^2 - r^4}{r^2}$$

```
> sol:=solve(CC1=0,a);
```

$$sol := \frac{1}{4}r^2, -\frac{1}{4}r^2$$

```
>
```

**Third Step: (5,6)-normalization ----- (4,5,6) jet**

```
> restart:
```

```
readlib(mtaylor):A:=0:B:=0:C1:=0:C2:=0: J:=0:a:=p*r^2:
```

```
x:=(z+cz)/2:y:=(z-cz)/(2*I):
```

```
S:=1/(1+y)-1:
```

```
T:=x/(1+y):
```

```
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
```

```
S2:=int(diff(int(diff(S1,z),z),cz),cz):
```

```
T2:=int(diff(int(diff(T1,z),z),cz),cz):
```

```
S3:=2*S2:
T3:=8*T2:
k:=1:ck:=1: m:=1:
```

```
f01:= a:
F2:= f01: h21:= 1/2*r^4: g001:= 0: H101:= 0: h101:= 0: G3:= 0: g3:=
0: H4:= 0: h4:= 0: G11:= 2*r*f01: G001:= -r*f01: g11:= 1/2*r^3:
H21:= 2*r^2*f01: H02:= 1/2*r^4: f2:= -1/4*r^2: h02:= 2*r^2*f01:
F01:= 1/4*r^2:
```

```
G101:= -a^2+1/16*r^4: g101:= -1/2*r^2*a: G4:= 0: g4:= 0: H5:= 0:
h5:= 0: h011:= 0: H201:= -1/16*r*(16*a^2-r^4): H011:= -r^3*a: G21:=
-1/4*r^4+2*a^2: H12:= r^3*a: h31:= 2*r^3*a: g21:= 3/2*r^2*a: H31:=
-1/4*r^5+4*r*a^2: h201:= -1/2*r^3*a: f11:= 0: F001:= -1/4*r*a:
F11:= r*a: f001:= -1/32*(16*a^2-r^4)/r: F3:= 1/16*(16*a^2-r^4)/r:
g02:= -4*a^2+3/8*r^4: G02:= -1/2*r^2*a: f3:= -1/2*r*a: h12:=
3/8*r^5+2*r*a^2:
```

```
w2:=u2+I*(z*cz+B*z^2*cz^2+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+(E1-
I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
cw2:=u2-I*(z*cz+B*z^2*cz^2+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+(E1-
I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
w3:=u3+I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2+J*z^3*cz^3):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2+J*z^3*cz^3):
```

### Write a mapping

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3+
(F4+I*f4)*z^4+(F21+I*f21)*z^2*w2+(F02+I*f02)*w2^2+
(F101+I*f101)*z*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3+
(F4-I*f4)*cz^4+(F21-I*f21)*cz^2*cw2+(F02-I*f02)*cw2^2+(F101-
I*f101)*cz*cw3:

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3+
(G5+I*g5)*z^5+(G31+I*g31)*z^3*w2+(G12+I*g12)*z*w2^2+
```

$(G201+I*g201)*z^2*w3+(G011+I*g011)*w2*w3:$

cg:=

$r^2*m*cw2+$

$(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+$

$(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-I*g101)*cz*cw3+$

$(G5-I*g5)*cz^5+(G31-I*g31)*cz^3*cw2+(G12-I*g12)*cz*cw2^2+(G201-I*g201)*cz^2*cw3+(G011-I*g011)*cw2*cw3:$

h:=

$r^3*m^2*w3+$

$(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+$

$(H101+I*h101)*z*w3+$

$(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+$

$(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3+$

$(H6+I*h6)*z^6+(H41+I*h41)*z^4*w2+(H22+I*h22)*z^2*w2^2+$

$(H03+I*h03)*w2^3+(H301+I*h301)*z^3*w3+(H111+I*h111)*z*w2*m^2*w3+$

$(H002+I*h002)*w3^2:$

ch:=

$r^3*m^2*cw3+$

$(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-I*h101)*cz*cw3+$

$(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3+$

$(H6-I*h6)*cz^6+(H41-I*h41)*cz^4*cw2+(H22-I*h22)*cz^2*cw2^2+(H03-I*h03)*cw2^3+(H301-I*h301)*cz^3*cw3+(H111-I*h111)*cz*cw2*cw3+(H002-I*h002)*cw3^2:$

**Write the relations for the mapping.**

> e2:=subs([z=f,cz=cf],2\*S3)+I\*(g-cg):

e3:=subs([z=f,cz=cf],2\*T3)+I\*(h-ch):

**Write the coefficients of the (5,6)-jet.**

>

P[5,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,z)))):

P[4,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,cz)))):

P[3,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,cz,cz)))):

P[3,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,u2)))):

P[2,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,u2)))):

P[1,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2,u2)))):

P[2,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u3)))):

```

))) :
P[1,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u3
)))) :
P[0,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u2,u3)
))):

P1[5,0,0,0]:=subs(I=0,P[5,0,0,0]):
P2[5,0,0,0]:=normal(expand((P[5,0,0,0]-P1[5,0,0,0])*I)):
P1[4,1,0,0]:=subs(I=0,P[4,1,0,0]):
P2[4,1,0,0]:=normal(expand((P[4,1,0,0]-P1[4,1,0,0])*I)):
P1[3,2,0,0]:=subs(I=0,P[3,2,0,0]):
P2[3,2,0,0]:=normal(expand((P[3,2,0,0]-P1[3,2,0,0])*I)):
P1[3,0,1,0]:=subs(I=0,P[3,0,1,0]):
P2[3,0,1,0]:=normal(expand((P[3,0,1,0]-P1[3,0,1,0])*I)):
P1[2,1,1,0]:=subs(I=0,P[2,1,1,0]):
P2[2,1,1,0]:=normal(expand((P[2,1,1,0]-P1[2,1,1,0])*I)):
P1[1,0,2,0]:=subs(I=0,P[1,0,2,0]):
P2[1,0,2,0]:=normal(expand((P[1,0,2,0]-P1[1,0,2,0])*I)):
P1[2,0,0,1]:=subs(I=0,P[2,0,0,1]):
P2[2,0,0,1]:=normal(expand((P[2,0,0,1]-P1[2,0,0,1])*I)):

Q[6,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z
,z,z)))) :
Q[5,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z
,z,cz)))) :
Q[4,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z
,cz,cz)))) :
Q[3,3,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,c
z,cz,cz)))) :
Q[4,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z
,u2)))) :
Q[3,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,c
z,u2)))) :
Q[2,2,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,
cz,u2)))) :
Q[2,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,
cz,u2)))) :
Q[1,1,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,
cz,u2)))) :
Q[0,0,3,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u2,u
2)))) :
Q[3,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u
3)))) :
Q[2,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,
u3)))) :
Q[1,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u3
)))) :
Q[0,0,0,2]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u3,u3)
))):

```

```

Q1[6,0,0,0]:=subs(I=0,Q[6,0,0,0]):
Q2[6,0,0,0]:=normal(expand((Q[6,0,0,0]-Q1[6,0,0,0])*I)):
Q1[5,1,0,0]:=subs(I=0,Q[5,1,0,0]):
Q2[5,1,0,0]:=normal(expand((Q[5,1,0,0]-Q1[5,1,0,0])*I)):
Q1[4,2,0,0]:=subs(I=0,Q[4,2,0,0]):
Q2[4,2,0,0]:=normal(expand((Q[4,2,0,0]-Q1[4,2,0,0])*I)):

Q1[4,0,1,0]:=subs(I=0,Q[4,0,1,0]):
Q2[4,0,1,0]:=normal(expand((Q[4,0,1,0]-Q1[4,0,1,0])*I)):
Q1[3,1,1,0]:=subs(I=0,Q[3,1,1,0]):
Q2[3,1,1,0]:=normal(expand((Q[3,1,1,0]-Q1[3,1,1,0])*I)):

Q1[2,0,2,0]:=subs(I=0,Q[2,0,2,0]):
Q2[2,0,2,0]:=normal(expand((Q[2,0,2,0]-Q1[2,0,2,0])*I)):

Q1[3,0,0,1]:=subs(I=0,Q[3,0,0,1]):
Q2[3,0,0,1]:=normal(expand((Q[3,0,0,1]-Q1[3,0,0,1])*I)):
Q1[2,1,0,1]:=subs(I=0,Q[2,1,0,1]):
Q2[2,1,0,1]:=normal(expand((Q[2,1,0,1]-Q1[2,1,0,1])*I)):
Q1[1,0,1,1]:=subs(I=0,Q[1,0,1,1]):
Q2[1,0,1,1]:=normal(expand((Q[1,0,1,1]-Q1[1,0,1,1])*I)):

```

**Write 39 equations.**

```

> EQ:={
P1[5,0,0,0]=0, P2[5,0,0,0]=0,
P1[4,1,0,0]=0, P2[4,1,0,0]=0,
P1[3,2,0,0]=0, P2[3,2,0,0]=0,
P1[3,0,1,0]=0, P2[3,0,1,0]=0,
P1[2,1,1,0]=0, P2[2,1,1,0]=0,
P1[1,0,2,0]=0, P2[1,0,2,0]=0,
P1[2,0,0,1]=0, P2[2,0,0,1]=0,
P[1,1,0,1]=0,
P[0,0,1,1]=0,

Q1[6,0,0,0]=0,Q2[6,0,0,0]=0,
Q1[5,1,0,0]=0,Q2[5,1,0,0]=0,
Q1[4,2,0,0]=0,Q2[4,2,0,0]=0,
Q[3,3,0,0]=0,
Q1[4,0,1,0]=0,Q2[4,0,1,0]=0,
Q1[3,1,1,0]=0,Q2[3,1,1,0]=0,
Q[2,2,1,0]=0,
Q1[2,0,2,0]=0,Q2[2,0,2,0]=0,
Q[1,1,2,0]=0,
Q[0,0,3,0]=0,
Q1[3,0,0,1]=0,Q2[3,0,0,1]=0,
Q1[2,1,0,1]=0,Q2[2,1,0,1]=0,

```



Q1[1,0,1,1]=0,Q2[1,0,1,1]=0,  
Q[0,0,0,2]=0  
}:

> sol:=solve(EQ)[2];

$$\text{sol} := \{F101 = -\frac{7}{4}p^2 r^4 + \frac{7}{64}r^4, E2 = -\frac{5}{96}r^3 + \frac{5}{6}p^2 r^3, g5 = 0, h6 = 0, H6 = 0, G5 = 0,$$

$$h002 = 0, D1 = 0, E1 = \frac{1}{6}r^3 p - \frac{8}{3}r^3 p^3, g011 = -\frac{1}{16}r^5 p - r^5 p^3,$$

$$g31 = -\frac{3}{32}r^5 + \frac{5}{2}r^5 p^2, f02 = \frac{17}{24}p^2 r^4 - \frac{5}{384}r^4, F02 = \frac{1}{12}r^4 p + \frac{2}{3}r^4 p^3,$$

$$h22 = \frac{9}{8}r^6 p + 6r^6 p^3, f21 = -\frac{5}{12}r^4 p + \frac{8}{3}r^4 p^3, h301 = -\frac{3}{2}r^6 p^2 + \frac{1}{32}r^6,$$

$$H03 = -\frac{9}{8}r^6 p + 14r^6 p^3, H22 = -\frac{13}{4}r^6 p^2 - \frac{3}{64}r^6, h41 = \frac{9}{2}r^6 p^2 - \frac{3}{32}r^6,$$

$$H41 = -\frac{9}{8}r^6 p + 6r^6 p^3, G31 = -\frac{7}{8}r^5 p + 2r^5 p^3, G12 = -\frac{13}{12}r^5 p^2 - \frac{23}{192}r^5,$$

$$G201 = \frac{5}{16}r^5 p - r^5 p^3, g12 = -\frac{1}{12}r^5 p + \frac{4}{3}r^5 p^3, G011 = -\frac{7}{4}r^5 p^2 + \frac{9}{64}r^5,$$

$$h03 = \frac{1}{2}r^6 p^2 + \frac{1}{32}r^6, H002 = -\frac{11}{8}r^6 p^2 + \frac{15}{128}r^6, D2 = 0, p = p, r = r,$$

$$H301 = \frac{3}{8}r^6 p - 2r^6 p^3, H111 = -\frac{3}{2}r^6 p^2 + \frac{1}{32}r^6, f101 = 0, f4 = -\frac{3}{4}p^2 r^4 + \frac{1}{64}r^4,$$

$$h111 = -\frac{5}{8}r^6 p - 2r^6 p^3, F4 = -\frac{3}{16}r^4 p + r^4 p^3, g201 = -r^5 p^2 + \frac{1}{32}r^5,$$

$$F21 = -\frac{2}{3}p^2 r^4 + \frac{5}{48}r^4\}$$

> EE1:=factor(1/6\*r^3\*p-8/3\*r^3\*p^3);

EE2:=factor(5/6\*p^2\*r^3-5/96\*r^3);

$$EE1 := -\frac{1}{6}r^3 p (4p - 1)(4p + 1)$$

$$EE2 := \frac{5}{96}r^3 (4p - 1)(4p + 1)$$

For  $p$  under consideration we get non-zero values, then **it is not the cubic for  $\gamma=0$ .**

## Type Ic for non-zero $\gamma$ .

First step: (3,4)-normalization -----

(2,3,4) jet

```
> restart:
```

```
A:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
```

**Original equations**

```
v_2 = x (1+y)^m + gamma((1+y)^(m+1)-1),
```

```
v_3 = (1+y)^m - 1.
```

**The origin is the center of the germ**

```
> T:= x*(1+y)^(m+n)*((1+y)^(m+1)-1): S:=(1+y)^m - 1:
```

**Load a package.**

```
> readlib(mttaylor):
```

**Take the Theilor (3,4)-degree expansion of F and G and bring the he standard form  $v_2=|z|^2+O(3)$ ,  $v_3=2\text{Re } k z^2 cz +O(4)$ .**

```
> S1:=mtaylor(4*S/m/(m-1), [z, cz], 8): T1:=mtaylor(4*T/n/m/(m+1), [z, cz], 9):
```

```
S2:=int(diff(int(diff(S1, z), z), cz), cz):
```

```
T2:=int(diff(int(diff(T1, z), z), cz), cz):
```

```
S3:=S2:
```

```
T3:=collect((T2-S3)*4*n*(m+1), [z, cz], distributed):
```

**Result check.**

```
> S[1,0,0,0]:=simplify(expand(subs([z=0, cz=0], diff(S3, z)))):
```

```
S[1,1,0,0]:=simplify(expand(subs([z=0, cz=0], diff(S3, z, cz)))):
```

```
S[2,0,0,0]:=simplify(expand(subs([z=0, cz=0], diff(S3, z, z)))):
```

```
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
```

```
T[1,0,0,0]:=simplify(expand(subs([z=0, cz=0], diff(T3, z)))):
```

```
T[1,1,0,0]:=simplify(expand(subs([z=0, cz=0], diff(T3, z, cz)))):
```

```
T[2,0,0,0]:=simplify(expand(subs([z=0, cz=0], diff(T3, z, z)))):
```

```
T[2,1,0,0]:=simplify(simplify(expand(subs([z=0, cz=0], diff(T3, z, z, cz))))/2):
```

```
T[3,0,0,0]:=simplify(expand(subs([z=0, cz=0], diff(T3, z, z, z)))):
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := m - I n m - 1 - I n$$

$$T_{3,0,0,0} := 0$$

### Introduce the notations.

> k:=m-I\*n\*m-1-I\*n: ck:=m+I\*n\*m-1+I\*n: M:=(m-1)^2+n^2\*(m+1)^2:

### Introduce new variables

> w2:=u2+I\*z\*cz: cw2:=u2-I\*z\*cz:

w3:=u3+I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):

cw3:=u3-I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):

### Write a mapping

> f:=

r\*ck\*z+

(F2+I\*f2)\*z^2+(F01+I\*f01)\*w2:

cf:=

r\*k\*cz+

(F2-I\*f2)\*cz^2+(F01-I\*f01)\*cw2:

g:=

r^2\*M\*w2+

(G3+I\*g3)\*z^3+(G11+I\*g11)\*z\*w2+(G001+I\*g001)\*w3:

cg:=

r^2\*M\*cw2+

(G3-I\*g3)\*cz^3+(G11-I\*g11)\*cz\*cw2+(G001-I\*g001)\*cw3:

h:=

r^3\*M^2\*w3+

(H4+I\*h4)\*z^4+(H21+I\*h21)\*z^2\*w2+(H02+I\*h02)\*w2^2+

(H101+I\*h101)\*z\*w3:

ch:=

r^3\*M^2\*cw3+

(H4-I\*h4)\*cz^4+(H21-I\*h21)\*cz^2\*cw2+(H02-I\*h02)\*cw2^2+(H101-

I\*h101)\*cz\*cw3:

### Write the relations for the mapping.

> e2:=subs([z=f,cz=cf],2\*S3)+I\*(g-cg):

e3:=subs([z=f,cz=cf],2\*T3)+I\*(h-ch):

### Write the coefficients of the (3,4)-jet.

> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):

```

P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):

P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):

Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**17 equations    Solve the system.**

```

> solve(EQ)[2];

```

$$\{h21 = -\frac{1}{6}(n^4 m^6 + m^6 + 2 n^2 m^6 - 6 n^2 m^5 - 7 m^5 + n^4 m^5 - 4 n^4 m^4 + 20 m^4 + 12 n^2 m^3 - 6 n^4 m^3 - 30 m^3 + 25 m^2 - 6 m^2 n^2 + n^4 m^2 - 11 m + 5 n^4 m - 6 n^2 m + 2 n^4 + 4 n^2 + 2) r^4, m = m, H101 = 0, h101 = 0, g001 = 0, H4 = 0, g3 = 0, G3 = 0, h4 = 0, G001 =$$

$$-\frac{1}{12} r (-2 n^4 r^2 - r^2 m^5 + 12 f2 + 6 r^2 m^4 - 14 r^2 m^3 + 16 r^2 m^2 - 9 r^2 m + 2 r^2 + 24 n^2 m f2 + 12 m^2 n^2 f2 + 2 n^4 r^2 m^4 + n^4 r^2 m^5 - 2 n^4 r^2 m^3 - 7 n^4 r^2 m - 8 n^4 r^2 m^2 + 12 m^2 f2 + 12 n^2 f2 - 24 m f2)/(n (m + 1)), f01 = \frac{1}{12} (-r^2 m^4 + r^2 m^4 n^2 - r^2 m^3 n^2$$

$$+ 5 r^2 m^3 - 3 r^2 n^2 m^2 - 9 r^2 m^2 + r^2 n^2 m + 7 r^2 m + 12 m f2 + 2 r^2 n^2 - 12 f2 - 2 r^2)/(n (m + 1)), F2 = \frac{1}{12} (-r^2 m^4 - r^2 m^4 n^2 + 5 r^2 m^3 + r^2 m^3 n^2 - 9 r^2 m^2 + 3 r^2 n^2 m^2 + 7 r^2 m + 12 m f2 - r^2 n^2 m - 12 f2 - 2 r^2 n^2 - 2 r^2)/(n (m + 1)), H21 = \frac{1}{6} r^2 ($$

$$-6 n^4 r^2 - 27 r^2 m^5 + 8 r^2 m^6 + 12 f2 + 50 r^2 m^4 - 55 r^2 m^3 + 36 r^2 m^2 - 13 r^2 m + 2 r^2 - r^2 m^7 + 12 m^4 f2 - 48 m^2 n^2 f2 - 6 n^4 r^2 m^4 - 15 n^4 r^2 m^5 + 21 n^4 r^2 m^3 - 9 n^4 r^2 m + 12 n^4 r^2 m^2 + 72 m^2 f2 - 48 m^3 f2 + 24 n^2 f2 + 3 n^4 r^2 m^7 + 24 n^2 m^4 f2 + 12 n^4 f2 + 6 n^6 m^5 r^2 - 20 n^6 m^4 r^2 - 50 n^6 m^3 r^2 - 48 n^6 m^2 r^2 - 22 n^6 m r^2 + 72 n^4 m^2 f2$$

$$+ 48 n^4 m f2 + 2 n^6 m^7 r^2 + 8 n^6 m^6 r^2 + 12 n^4 m^4 f2 + 48 n^4 m^3 f2 - 4 n^6 r^2 - 48 m f2)/(n (m + 1)), H02 = -\frac{1}{6}(n^4 m^6 + m^6 + 2 n^2 m^6 - 6 n^2 m^5 - 7 m^5 + n^4 m^5 - 4 n^4 m^4 + 20 m^4 + 12 n^2 m^3 - 6 n^4 m^3 - 30 m^3 + 25 m^2 - 6 m^2 n^2 + n^4 m^2 - 11 m + 5 n^4 m$$

$$- 6 n^2 m + 2 n^4 + 4 n^2 + 2) r^4,$$

$$g11 = -\frac{1}{6}(m^4 n^2 + m^4 - 5 m^3 - n^2 m^3 + 9 m^2 - 3 m^2 n^2 - 7 m + n^2 m + 2 n^2 + 2) r^3,$$

$$G11 = \frac{1}{6} r (-4 n^4 r^2 - r^2 m^5 + 12 f2 - 2 r^2 n^2 + 6 r^2 m^4 - 14 r^2 m^3 + 16 r^2 m^2 - 9 r^2 m$$

$$\begin{aligned}
& + 2r^2 + 24n^2mf2 + 12m^2n^2f2 + 4n^4r^2m^4 + 2n^4r^2m^5 + n^2r^2m^5 - 4n^4r^2m^3 \\
& - 14n^4r^2m - 16n^4r^2m^2 + 12m^2f2 + 12n^2f2 + 4r^2n^2m^2 - 24mf2 - 2r^2m^3n^2 \\
& - 2r^2m^4n^2 + r^2n^2m)/(n(m+1)), F01 = -\frac{1}{6}r^2m^3n^2 + \frac{1}{2}r^2n^2m + \frac{1}{3}r^2n^2 - f2, h02
\end{aligned}$$

$$\begin{aligned}
& = \frac{1}{6}r^2(-2n^4r^2 - 27r^2m^5 + 8r^2m^6 + 12f2 + 2r^2n^2 + 50r^2m^4 - 55r^2m^3 + 36r^2m^2 \\
& - 13r^2m + 2r^2 - r^2m^7 + 12m^4f2 - 48m^2n^2f2 - 2n^4r^2m^4 - 5n^4r^2m^5 - 3n^2r^2m^5 \\
& + 4n^2r^2m^6 + 7n^4r^2m^3 - 3n^4r^2m + 4n^4r^2m^2 + 72m^2f2 - 48m^3f2 + 24n^2f2
\end{aligned}$$

$$\begin{aligned}
& + n^4r^2m^7 + 24n^2m^4f2 + 12n^4f2 + 3n^6m^5r^2 - 10n^6m^4r^2 - 25n^6m^3r^2 \\
& - 24n^6m^2r^2 - 11n^6mr^2 + 72n^4m^2f2 + 48n^4mf2 + n^6m^7r^2 + 4n^6m^6r^2 \\
& + 12n^4m^4f2 + 48n^4m^3f2 - m^7r^2n^2 - 2n^6r^2 - 48mf2 + 9r^2m^3n^2 - 6r^2m^4n^2 \\
& - 5r^2n^2m)/(n(m+1)), r = r, f2 = f2, n = n \}
\end{aligned}$$

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```

> restart:
readlib(mtaylor):
A:=0:B:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
T:= x*(1+y)^m+n*((1+y)^(m+1)-1): S:=(1+y)^m-1:
S1:=mtaylor(4*S/m/(m-1),[z,cz],8): T1:=mtaylor(4*T/n/m/(m+1),
[z,cz],9):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=collect((T2-S3)*4*n*(m+1),[z,cz],distributed):
k:=m-I*n*m-1-I*n:ck:=m+I*n*m-1+I*n: M:=(m-1)^2+n^2*(m+1)^2:
> f2:=a:
h02:= 1/6*r^2*(-27*r^2*m^5+8*r^2*m^6-2*n^4*r^2+12*f2+2*r^2*n^2-
55*r^2*m^3+36*r^2*m^2+50*r^2*m^4-13*r^2*m+12*m^4*f2-
r^2*m^7+24*n^2*f2-48*m^3*f2+72*m^2*f2-48*m^2*n^2*f2+2*r^2-
5*n^4*r^2*m^5-3*n^2*r^2*m^5-
2*n^4*r^2*m^4+7*n^4*r^2*m^3+4*n^4*r^2*m^2-
3*n^4*r^2*m+4*n^2*r^2*m^6-48*m*f2-5*n^2*r^2*m-
6*r^2*m^4*n^2+9*n^2*r^2*m^3-
2*n^6*r^2+12*n^4*f2+n^6*m^7*r^2+4*n^6*m^6*r^2+3*n^6*m^5*r^2-
10*n^6*m^4*r^2-25*n^6*m^3*r^2+12*n^4*m^4*f2+48*n^4*m^3*f2-
24*n^6*m^2*r^2+72*n^4*m^2*f2-n^2*m^7*r^2-
11*n^6*m*r^2+48*n^4*m*f2+n^4*m^7*r^2+24*n^2*m^4*f2)/(n*(m+1)):
F01:= -1/6*n^2*r^2*m^3+1/2*n^2*r^2*m+1/3*r^2*n^2-f2:
H21:= 1/6*r^2*(-27*r^2*m^5+8*r^2*m^6-6*n^4*r^2+12*f2-

```

```

55*r^2*m^3+36*r^2*m^2+50*r^2*m^4-13*r^2*m+12*m^4*f2-
r^2*m^7+24*n^2*f2-48*m^3*f2+72*m^2*f2-48*m^2*n^2*f2+2*r^2-
15*n^4*r^2*m^5-6*n^4*r^2*m^4+21*n^4*r^2*m^3+12*n^4*r^2*m^2-
9*n^4*r^2*m-48*m*f2-
4*n^6*r^2+12*n^4*f2+2*n^6*m^7*r^2+8*n^6*m^6*r^2+6*n^6*m^5*r^2-
20*n^6*m^4*r^2-50*n^6*m^3*r^2+12*n^4*m^4*f2+48*n^4*m^3*f2-
48*n^6*m^2*r^2+72*n^4*m^2*f2-
22*n^6*m*r^2+48*n^4*m*f2+3*n^4*m^7*r^2+24*n^2*m^4*f2) / (n*(m+1)) :
H02:= -1/6*(m^6+2*n^2*m^6+n^4*m^6+n^4*m^5-6*n^2*m^5-7*m^5+20*m^4-
4*n^4*m^4-6*n^4*m^3-30*m^3+12*n^2*m^3+25*m^2+n^4*m^2-6*n^2*m^2-
11*m+5*n^4*m-6*n^2*m+2*n^4+4*n^2+2)*r^4:
G3:= 0: g3:= 0: H4:= 0: h4:= 0: h101:= 0: H101:= 0: g001:= 0:
g11:= -1/6*(m^4+m^4*n^2-5*m^3-n^2*m^3+9*m^2-3*n^2*m^2+n^2*m-
7*m+2*n^2+2)*r^3:
h21:=-1/6*(m^6+2*n^2*m^6+n^4*m^6+n^4*m^5-6*n^2*m^5-7*m^5+20*m^4-
4*n^4*m^4-6*n^4*m^3-30*m^3+12*n^2*m^3+25*m^2+n^4*m^2-6*n^2*m^2-
11*m+5*n^4*m-6*n^2*m+2*n^4+4*n^2+2)*r^4:
G11:=1/6*r*(-r^2*m^5-4*n^4*r^2+12*f2-2*r^2*n^2-
14*r^2*m^3+16*r^2*m^2+6*r^2*m^4-
9*r^2*m+12*n^2*f2+12*m^2*f2+24*n^2*m*f2+12*m^2*n^2*f2+2*r^2+2*n^4*r
^2*m^5+n^2*r^2*m^5+4*n^4*r^2*m^4-4*n^4*r^2*m^3-16*n^4*r^2*m^2-
14*n^4*r^2*m-24*m*f2+4*r^2*n^2*m^2+n^2*r^2*m-2*r^2*m^4*n^2-
2*n^2*r^2*m^3) / (n*(m+1)) :
G001:= -1/12*r*(-r^2*m^5-2*n^4*r^2+12*f2-
14*r^2*m^3+16*r^2*m^2+6*r^2*m^4-
9*r^2*m+12*n^2*f2+12*m^2*f2+24*n^2*m*f2+12*m^2*n^2*f2+2*r^2+n^4*r^2
*m^5+2*n^4*r^2*m^4-2*n^4*r^2*m^3-8*n^4*r^2*m^2-7*n^4*r^2*m-
24*m*f2) / (n*(m+1)) :
f01:=1/12*(r^2*m^4*n^2-r^2*m^4+5*r^2*m^3-n^2*r^2*m^3-3*r^2*n^2*m^2-
9*r^2*m^2+n^2*r^2*m+7*r^2*m+12*m*f2+2*r^2*n^2-2*r^2-12*f2) /
(n*(m+1)) :
F2:= -1/12*(r^2*m^4*n^2+r^2*m^4-5*r^2*m^3-n^2*r^2*m^3-
3*r^2*n^2*m^2+9*r^2*m^2-7*r^2*m-
12*m*f2+n^2*r^2*m+2*r^2+12*f2+2*r^2*n^2) / (n*(m+1)) :

```

### Introduce new variables

```

> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):

```

### Write a mapping

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

```

```

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

```

```

g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

```

```

cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

```

```

h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

```

```

ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
```

```
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (4,5)-jet.**

```

> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):

```

```
P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
```

```
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
```

```
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
```

```
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):
```



```

Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write the equations and the variables.**

```

> X:={
F3,f3,F11,f11,F001,f001,G4,g4,G21,g21,G02,g02,G101,g101,H5,h5,H31,h
31,H12,h12,H201,h201,H011,h011,C1,C2}:
> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Number of equations - 26.**

```
> sol:=solve(EQ,X):
> factor(sol[4]);
```

$$C2 = \frac{1}{108} (m - 1)(m - 2)(2m - 1)(m + 1) n r^2$$

```
>
```

So it is not the cubic.

**Type 1c for gamma=0.**

First step: (3,4)-normalization -----

(2,3,4) jet

```
> restart:readlib(mttaylor):
n:=0:A:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
T:= x*(1+y)^m: S:=(1+y)^m -1:
S1:=mtaylor(4*S/m/(m-1),[z,cz],8): T1:=mtaylor(16*T/m,[z,cz],9):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=T2:
```

**Result check.**

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)
))))/2);
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := m - 1$$

$$T_{3,0,0,0} := 0$$

### Introduce the notations.

```
> k:=m-1:ck:=m-1: M:=(m-1)^2:
```

### Introduce new variables

```
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):
```

### Write a mapping

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:
```

```
cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:
```

```
g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

### Write the relations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

### Write the coefficients of the (3,4)-jet.

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```

P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):

```

```

Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

```

```

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

```

```

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**17 equations      Solve the system.**

```

> solve(EQ)[3];

```

$$\{g11 = -\frac{1}{6}r^3 m^4 + \frac{5}{6}r^3 m^3 - \frac{3}{2}r^3 m^2 + \frac{7}{6}r^3 m - \frac{1}{3}r^3, G001 = -r f01 m + r f01, h101 = 0,$$

$$G3 = 0, g001 = 0, H101 = 0, g3 = 0, H4 = 0, h4 = 0,$$

$$F01 = -\frac{1}{12}r^2 m^3 + \frac{1}{3}r^2 m^2 - \frac{5}{12}r^2 m + \frac{1}{6}r^2, f2 = \frac{1}{12}r^2 m^3 - \frac{1}{3}r^2 m^2 + \frac{5}{12}r^2 m - \frac{1}{6}r^2,$$

$$\begin{aligned}
f01 &= f01, m = m, h02 = 2 r^2 m^3 f01 - 6 r^2 m^2 f01 + 6 r^2 f01 m - 2 r^2 f01, F2 = f01, \\
G11 &= 2 r f01 m - 2 r f01, \\
h21 &= -\frac{1}{6} r^4 m^6 + \frac{7}{6} r^4 m^5 - \frac{10}{3} r^4 m^4 + 5 r^4 m^3 - \frac{25}{6} r^4 m^2 + \frac{11}{6} r^4 m - \frac{1}{3} r^4, \\
H21 &= 2 r^2 m^3 f01 - 6 r^2 m^2 f01 + 6 r^2 f01 m - 2 r^2 f01, r = r, \\
H02 &= -\frac{1}{6} r^4 m^6 + \frac{7}{6} r^4 m^5 - \frac{10}{3} r^4 m^4 + 5 r^4 m^3 - \frac{25}{6} r^4 m^2 + \frac{11}{6} r^4 m - \frac{1}{3} r^4
\end{aligned}$$

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```

> restart:readlib(mtaylor):
n:=0:A:=0:B:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
T:= x*(1+y)^m: S:=(1+y)^m -1:
S1:=mtaylor(4*S/m/(m-1), [z, cz], 8): T1:=mtaylor(16*T/m, [z, cz], 9):
S2:=int(diff(int(diff(S1, z), z), cz), cz):
T2:=int(diff(int(diff(T1, z), z), cz), cz):
S3:=S2:
T3:=T2:
k:=m-1:ck:=m-1: M:=(m-1)^2:
> f01:=a:
g11:= -1/6*r^3*m^4+5/6*r^3*m^3-3/2*r^3*m^2+7/6*r^3*m-1/3*r^3:
G001:= -r*f01*m+r*f01: h101:= 0: G3:= 0: g001:= 0: H101:= 0: g3:=
0: H4:= 0: h4:= 0: F01:= -1/12*r^2*m^3+1/3*r^2*m^2-
5/12*r^2*m+1/6*r^2: f2:= 1/12*r^2*m^3-1/3*r^2*m^2+5/12*r^2*m-
1/6*r^2: h02:= 2*r^2*m^3*f01-6*r^2*m^2*f01+6*r^2*f01*m-2*r^2*f01:
F2:= f01: G11:= 2*r*f01*m-2*r*f01: h21:= -1/6*r^4*m^6+7/6*r^4*m^5-
10/3*r^4*m^4+5*r^4*m^3-25/6*r^4*m^2+11/6*r^4*m-1/3*r^4: H21:=
2*r^2*m^3*f01-6*r^2*m^2*f01+6*r^2*f01*m-2*r^2*f01: H02:=
-1/6*r^4*m^6+7/6*r^4*m^5-10/3*r^4*m^4+5*r^4*m^3-
25/6*r^4*m^2+11/6*r^4*m-1/3*r^4:

```

### Introduce new variables

```

> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):

```

### Write a mapping

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

```

```

g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

```

```

cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

```

```

h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

```

```

ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

**Write the relations for the mapping.**

```

> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):

```

**Write the coefficients of the (4,5)-jet.**

```

> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):

```

```

P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):

```

```

Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):

```

```

Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write the equations and the variables.**

```

> X:={
F3,f3,F11,f11,F001,f001,G4,g4,G21,g21,G02,g02,G101,g101,H5,h5,H31,h
31,H12,h12,H201,h201,H011,h011,C1,C2}:
> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Number of equations - 26.**

```
> sol:=solve(EQ)[2];
```

$$\begin{aligned} \text{sol} := \{ & h12 = 2 r a^2 - 4 r m a^2 + \frac{43}{24} r^5 m^6 - \frac{13}{3} r^5 m^5 - \frac{7}{6} r^5 m + \frac{85}{24} r^5 m^2 + \frac{1}{24} r^5 m^8 \\ & + \frac{155}{24} r^5 m^4 - \frac{5}{12} r^5 m^7 - \frac{73}{12} r^5 m^3 + \frac{1}{6} r^5 + 2 r m^2 a^2, \end{aligned}$$

$$H12 = \frac{2}{3} r^3 a - \frac{14}{3} r^3 m^3 a + \frac{16}{3} r^3 m^2 a - 3 r^3 m a - \frac{1}{3} r^3 m^5 a + 2 r^3 m^4 a,$$

$$g21 = -\frac{1}{2} r^2 m^3 a + 2 r^2 m^2 a - \frac{5}{2} r^2 a m + r^2 a,$$

$$G21 = -\frac{1}{36} r^4 m^6 + \frac{2}{9} r^4 m^5 - \frac{13}{18} r^4 m^4 + \frac{11}{9} r^4 m^3 - \frac{41}{36} r^4 m^2 + \frac{5}{9} r^4 m - \frac{1}{9} r^4 + 2 a^2,$$

$$f11 = 0, h31 = -6 r^3 m a + \frac{4}{3} r^3 a - \frac{2}{3} r^3 m^5 a + \frac{32}{3} r^3 m^2 a - \frac{28}{3} r^3 m^3 a + 4 r^3 m^4 a,$$

$$G4 = 0, g4 = 0, h5 = 0, h011 = 0, H5 = 0, a = a, f001 =$$

$$\frac{1}{288} \frac{2 r^4 m^6 - 13 r^4 m^5 + 34 r^4 m^4 - 46 r^4 m^3 + 34 r^4 m^2 - 13 r^4 m - 144 a^2 + 2 r^4}{r(m-1)},$$

$$f3 = \frac{1}{6} r m^2 a - \frac{1}{2} r a m + \frac{1}{3} r a, CI =$$

$$-\frac{1}{24} \frac{2 r^4 m^6 - 13 r^4 m^5 + 34 r^4 m^4 - 46 r^4 m^3 + 34 r^4 m^2 - 13 r^4 m - 144 a^2 + 2 r^4}{r^2 (m-1)^2}, F3$$

$$= -\frac{1}{144} \frac{2 r^4 m^6 - 13 r^4 m^5 + 34 r^4 m^4 - 46 r^4 m^3 + 34 r^4 m^2 - 13 r^4 m - 144 a^2 + 2 r^4}{r(m-1)},$$

$$g02 =$$

$$\frac{11}{144} r^4 m^6 - \frac{73}{144} r^4 m^5 + \frac{49}{36} r^4 m^4 - \frac{137}{72} r^4 m^3 + \frac{211}{144} r^4 m^2 - \frac{85}{144} r^4 m + \frac{7}{72} r^4 - 4 a^2,$$

$$H201 = \frac{1}{144} (m^2 - 2 m + 1) r$$

$$(2 r^4 m^6 - 13 r^4 m^5 + 34 r^4 m^4 - 46 r^4 m^3 + 34 r^4 m^2 - 13 r^4 m - 144 a^2 + 2 r^4),$$



$$h201 = \frac{1}{6} r^3 m^5 a - r^3 m^4 a + \frac{7}{3} r^3 m^3 a - \frac{8}{3} r^3 m^2 a + \frac{3}{2} r^3 m a - \frac{1}{3} r^3 a, C2 = 0,$$

$$F11 = -\frac{1}{3} r m^2 a + r a m - \frac{2}{3} r a,$$

$$G101 = \frac{1}{72} r^4 m^6 - \frac{13}{144} r^4 m^5 + \frac{17}{72} r^4 m^4 - \frac{23}{72} r^4 m^3 + \frac{17}{72} r^4 m^2 - \frac{13}{144} r^4 m - a^2 + \frac{1}{72} r^4,$$

$$G02 = -\frac{1}{3} r^2 a - \frac{2}{3} r^2 m^2 a + \frac{1}{6} r^2 m^3 a + \frac{5}{6} r^2 a m, F001 = \frac{1}{12} r m^2 a - \frac{1}{4} r a m + \frac{1}{6} r a,$$

$$H011 = \frac{1}{3} r^3 m^5 a - 2 r^3 m^4 a + \frac{14}{3} r^3 m^3 a - \frac{16}{3} r^3 m^2 a + 3 r^3 m a - \frac{2}{3} r^3 a, r = r, H31$$

$$= 4 r a^2 - 8 r m a^2 + 4 r m^2 a^2 - \frac{1}{9} r^5 + \frac{5}{18} r^5 m^7 + \frac{73}{18} r^5 m^3 + \frac{7}{9} r^5 m - \frac{85}{36} r^5 m^2$$

$$- \frac{155}{36} r^5 m^4 + \frac{26}{9} r^5 m^5 - \frac{43}{36} r^5 m^6 - \frac{1}{36} r^5 m^8,$$

$$g101 = -\frac{1}{3} r^2 a - \frac{2}{3} r^2 m^2 a + \frac{1}{6} r^2 m^3 a + \frac{5}{6} r^2 a m, m = m \}$$

> CC1 := factor (1/24 \* (-2\*r^4\*m^6+13\*r^4\*m^5-34\*r^4\*m^4+46\*r^4\*m^3-34\*r^4\*m^2+13\*r^4\*m-2\*r^4+144\*a^2) / (r^2\*(m-1)^2) );

$$CCI := \frac{1}{24} \frac{-2 r^4 m^6 + 13 r^4 m^5 - 34 r^4 m^4 + 46 r^4 m^3 - 34 r^4 m^2 + 13 r^4 m - 2 r^4 + 144 a^2}{r^2 (m-1)^2}$$

### Third Step: (5,6) - normalization ----- (4,5,6) jet.

> restart: readlib (mtaylor) :

n:=0:A:=0:B:=0:C2:=0:

C1:= 1/24 \* (-2\*r^4\*m^6+13\*r^4\*m^5-34\*r^4\*m^4+46\*r^4\*m^3-34\*r^4\*m^2+13\*r^4\*m-2\*r^4+144\*a^2) / (r^2\*(m-1)^2) : J:=0:

x:=(z+cz)/2:y:=(z-cz)/(2\*I):

T:= x\*(1+y)^m: S:=(1+y)^m -1:

S1:=mtaylor(4\*S/m/(m-1), [z, cz], 8) : T1:=mtaylor(16\*T/m, [z, cz], 9) :

S2:=int(diff(int(diff(S1, z), z), cz), cz) :

T2:=int(diff(int(diff(T1, z), z), cz), cz) :

S3:=S2:

T3:=T2:

k:=m-1:ck:=m-1: M:=(m-1)^2:

f01:=a:

g11:= -1/6\*r^3\*m^4+5/6\*r^3\*m^3-3/2\*r^3\*m^2+7/6\*r^3\*m-1/3\*r^3:

G001:= -r\*f01\*m+r\*f01: h101:= 0: G3:= 0: g001:= 0: H101:= 0: g3:=

```

0: H4:= 0: h4:= 0: F01:= -1/12*r^2*m^3+1/3*r^2*m^2-
5/12*r^2*m+1/6*r^2: f2:= 1/12*r^2*m^3-1/3*r^2*m^2+5/12*r^2*m-
1/6*r^2: h02:= 2*r^2*m^3*f01-6*r^2*m^2*f01+6*r^2*f01*m-2*r^2*f01:
F2:= f01: G11:= 2*r*f01*m-2*r*f01: h21:= -1/6*r^4*m^6+7/6*r^4*m^5-
10/3*r^4*m^4+5*r^4*m^3-25/6*r^4*m^2+11/6*r^4*m-1/3*r^4: H21:=
2*r^2*m^3*f01-6*r^2*m^2*f01+6*r^2*f01*m-2*r^2*f01: H02:=
-1/6*r^4*m^6+7/6*r^4*m^5-10/3*r^4*m^4+5*r^4*m^3-
25/6*r^4*m^2+11/6*r^4*m-1/3*r^4:
> h12:= 2*r*a^2-4*r*m*a^2+43/24*r^5*m^6-13/3*r^5*m^5-
7/6*r^5*m+85/24*r^5*m^2+1/24*r^5*m^8+155/24*r^5*m^4-5/12*r^5*m^7-
73/12*r^5*m^3+1/6*r^5+2*r*m^2*a^2: H12:= 2/3*r^3*a-
14/3*r^3*m^3*a+16/3*r^3*m^2*a-3*r^3*m*a-1/3*r^3*m^5*a+2*r^3*m^4*a:
g21:= -1/2*r^2*m^3*a+2*r^2*m^2*a-5/2*r^2*a*m+r^2*a: G21:=
-1/36*r^4*m^6+2/9*r^4*m^5-13/18*r^4*m^4+11/9*r^4*m^3-
41/36*r^4*m^2+5/9*r^4*m-1/9*r^4+2*a^2: f11:= 0: h31:=
-6*r^3*m*a+4/3*r^3*a-2/3*r^3*m^5*a+32/3*r^3*m^2*a-
28/3*r^3*m^3*a+4*r^3*m^4*a: G4:= 0: g4:= 0: h5:= 0: h011:= 0: H5:=
0:
f001:= 1/288*(2*r^4*m^6-13*r^4*m^5+34*r^4*m^4-
46*r^4*m^3+34*r^4*m^2-13*r^4*m-144*a^2+2*r^4)/(r*(m-1)): f3:=
1/6*r*m^2*a-1/2*r*a*m+1/3*r*a: F3:= -1/144*(2*r^4*m^6-
13*r^4*m^5+34*r^4*m^4-46*r^4*m^3+34*r^4*m^2-13*r^4*m-
144*a^2+2*r^4)/(r*(m-1)): g02:= 11/144*r^4*m^6-
73/144*r^4*m^5+49/36*r^4*m^4-137/72*r^4*m^3+211/144*r^4*m^2-
85/144*r^4*m+7/72*r^4-4*a^2: H201:= 1/144*(m^2-2*m+1)*r*(2*r^4*m^6-
13*r^4*m^5+34*r^4*m^4-46*r^4*m^3+34*r^4*m^2-13*r^4*m-
144*a^2+2*r^4): h201:= 1/6*r^3*m^5*a-r^3*m^4*a+7/3*r^3*m^3*a-
8/3*r^3*m^2*a+3/2*r^3*m*a-1/3*r^3*a: F11:= -1/3*r*m^2*a+r*a*m-
2/3*r*a: G101:= 1/72*r^4*m^6-13/144*r^4*m^5+17/72*r^4*m^4-
23/72*r^4*m^3+17/72*r^4*m^2-13/144*r^4*m-a^2+1/72*r^4: G02:=
-1/3*r^2*a-2/3*r^2*m^2*a+1/6*r^2*m^3*a+5/6*r^2*a*m: F001:=
1/12*r*m^2*a-1/4*r*a*m+1/6*r*a: H011:= 1/3*r^3*m^5*a-
2*r^3*m^4*a+14/3*r^3*m^3*a-16/3*r^3*m^2*a+3*r^3*m*a-2/3*r^3*a:
H31:= 4*r*a^2-8*r*m*a^2+4*r*m^2*a^2-
1/9*r^5+5/18*r^5*m^7+73/18*r^5*m^3+7/9*r^5*m-85/36*r^5*m^2-
155/36*r^5*m^4+26/9*r^5*m^5-43/36*r^5*m^6-1/36*r^5*m^8: g101:=
-1/3*r^2*a-2/3*r^2*m^2*a+1/6*r^2*m^3*a+5/6*r^2*a*m:

```

### Write a mapping

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3+
(F4+I*f4)*z^4+(F21+I*f21)*z^2*w2+(F02+I*f02)*w2^2+
(F101+I*f101)*z*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3+

```

$(F4-I*f4)*cz^4+(F21-I*f21)*cz^2*cw2+(F02-I*f02)*cw2^2+(F101-I*f101)*cz*cw3:$

$g:=$   
 $r^2*M*w2+$   
 $(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+$   
 $(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+$   
 $(G101+I*g101)*z*w3+$   
 $(G5+I*g5)*z^5+(G31+I*g31)*z^3*w2+(G12+I*g12)*z*w2^2+$   
 $(G201+I*g201)*z^2*w3+(G011+I*g011)*w2*w3:$

$cg:=$   
 $r^2*M*cw2+$   
 $(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+$   
 $(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-I*g101)*cz*cw3+$   
 $(G5-I*g5)*cz^5+(G31-I*g31)*cz^3*cw2+(G12-I*g12)*cz*cw2^2+(G201-I*g201)*cz^2*cw3+(G011-I*g011)*cw2*cw3:$

$h:=$   
 $r^3*M^2*w3+$   
 $(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+$   
 $(H101+I*h101)*z*w3+$   
 $(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+$   
 $(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3+$   
 $(H6+I*h6)*z^6+(H41+I*h41)*z^4*w2+(H22+I*h22)*z^2*w2^2+$   
 $(H03+I*h03)*w2^3+(H301+I*h301)*z^3*w3+(H111+I*h111)*z*w2*w3+$   
 $(H002+I*h002)*w3^2:$

$ch:=$   
 $r^3*M^2*cw3+$   
 $(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-I*h101)*cz*cw3+$   
 $(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3+$   
 $(H6-I*h6)*cz^6+(H41-I*h41)*cz^4*cw2+(H22-I*h22)*cz^2*cw2^2+(H03-I*h03)*cw2^3+(H301-I*h301)*cz^3*cw3+(H111-I*h111)*cz*cw2*cw3+(H002-I*h002)*cw3^2:$

$> w2:=u2+I*(z*cz+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+(E1-I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):$   
 $cw2:=u2-I*(z*cz+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+(E1-I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):$

$w3:=u3+I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):$   
 $cw3:=u3-I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):$

**Write the relations for the mapping.**

$> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):$

e3:=subs ([z=f,cz=cf],2\*T3)+I\*(h-ch):

**Write the coefficients of the (5,6)-jet.**

>

```
P[5,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,z)))):
P[4,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,cz)))):
P[3,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,cz,cz)))):
P[3,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,u2)))):
P[2,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,u2)))):
P[1,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2,u2)))):
P[2,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u3)))):
P[1,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u3)))):
P[0,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u2,u3)))):
```

```
P1[5,0,0,0]:=subs(I=0,P[5,0,0,0]):
P2[5,0,0,0]:=normal(expand((P[5,0,0,0]-P1[5,0,0,0])*I)):
P1[4,1,0,0]:=subs(I=0,P[4,1,0,0]):
P2[4,1,0,0]:=normal(expand((P[4,1,0,0]-P1[4,1,0,0])*I)):
P1[3,2,0,0]:=subs(I=0,P[3,2,0,0]):
P2[3,2,0,0]:=normal(expand((P[3,2,0,0]-P1[3,2,0,0])*I)):
P1[3,0,1,0]:=subs(I=0,P[3,0,1,0]):
P2[3,0,1,0]:=normal(expand((P[3,0,1,0]-P1[3,0,1,0])*I)):
P1[2,1,1,0]:=subs(I=0,P[2,1,1,0]):
P2[2,1,1,0]:=normal(expand((P[2,1,1,0]-P1[2,1,1,0])*I)):
P1[1,0,2,0]:=subs(I=0,P[1,0,2,0]):
P2[1,0,2,0]:=normal(expand((P[1,0,2,0]-P1[1,0,2,0])*I)):
P1[2,0,0,1]:=subs(I=0,P[2,0,0,1]):
P2[2,0,0,1]:=normal(expand((P[2,0,0,1]-P1[2,0,0,1])*I)):
```

```
Q[6,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z,z,z)))):
Q[5,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z,cz)))):
Q[4,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,cz,cz)))):
Q[3,3,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz,cz,cz)))):
Q[4,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,u2)))):
Q[3,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz,u2)))):
```

```
Q[2,2,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz,u2))))):
```

```
Q[2,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2,u2))))):
```

```
Q[1,1,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2,u2))))):
```

```
Q[0,0,3,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u2,u2))))):
```

```
Q[3,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u3))))):
```

```
Q[2,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u3))))):
```

```
Q[1,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u3))))):
```

```
Q[0,0,0,2]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u3,u3))))):
```

```
Q1[6,0,0,0]:=subs(I=0,Q[6,0,0,0]):
```

```
Q2[6,0,0,0]:=normal(expand((Q[6,0,0,0]-Q1[6,0,0,0])*I)):
```

```
Q1[5,1,0,0]:=subs(I=0,Q[5,1,0,0]):
```

```
Q2[5,1,0,0]:=normal(expand((Q[5,1,0,0]-Q1[5,1,0,0])*I)):
```

```
Q1[4,2,0,0]:=subs(I=0,Q[4,2,0,0]):
```

```
Q2[4,2,0,0]:=normal(expand((Q[4,2,0,0]-Q1[4,2,0,0])*I)):
```

```
Q1[4,0,1,0]:=subs(I=0,Q[4,0,1,0]):
```

```
Q2[4,0,1,0]:=normal(expand((Q[4,0,1,0]-Q1[4,0,1,0])*I)):
```

```
Q1[3,1,1,0]:=subs(I=0,Q[3,1,1,0]):
```

```
Q2[3,1,1,0]:=normal(expand((Q[3,1,1,0]-Q1[3,1,1,0])*I)):
```

```
Q1[2,0,2,0]:=subs(I=0,Q[2,0,2,0]):
```

```
Q2[2,0,2,0]:=normal(expand((Q[2,0,2,0]-Q1[2,0,2,0])*I)):
```

```
Q1[3,0,0,1]:=subs(I=0,Q[3,0,0,1]):
```

```
Q2[3,0,0,1]:=normal(expand((Q[3,0,0,1]-Q1[3,0,0,1])*I)):
```

```
Q1[2,1,0,1]:=subs(I=0,Q[2,1,0,1]):
```

```
Q2[2,1,0,1]:=normal(expand((Q[2,1,0,1]-Q1[2,1,0,1])*I)):
```

```
Q1[1,0,1,1]:=subs(I=0,Q[1,0,1,1]):
```

```
Q2[1,0,1,1]:=normal(expand((Q[1,0,1,1]-Q1[1,0,1,1])*I)):
```

### Write 39 equations.

```
> EQ:={
```

```
P1[5,0,0,0]=0, P2[5,0,0,0]=0,
```

```
P1[4,1,0,0]=0, P2[4,1,0,0]=0,
```

```
P1[3,2,0,0]=0, P2[3,2,0,0]=0,
```

```
P1[3,0,1,0]=0, P2[3,0,1,0]=0,
```

$P1[2, 1, 1, 0]=0, P2[2, 1, 1, 0]=0,$   
 $P1[1, 0, 2, 0]=0, P2[1, 0, 2, 0]=0,$   
 $P1[2, 0, 0, 1]=0, P2[2, 0, 0, 1]=0,$   
 $P[1, 1, 0, 1]=0,$   
 $P[0, 0, 1, 1]=0,$

$Q1[6, 0, 0, 0]=0, Q2[6, 0, 0, 0]=0,$   
 $Q1[5, 1, 0, 0]=0, Q2[5, 1, 0, 0]=0,$   
 $Q1[4, 2, 0, 0]=0, Q2[4, 2, 0, 0]=0,$   
 $Q[3, 3, 0, 0]=0,$   
 $Q1[4, 0, 1, 0]=0, Q2[4, 0, 1, 0]=0,$   
 $Q1[3, 1, 1, 0]=0, Q2[3, 1, 1, 0]=0,$   
 $Q[2, 2, 1, 0]=0,$   
 $Q1[2, 0, 2, 0]=0, Q2[2, 0, 2, 0]=0,$   
 $Q[1, 1, 2, 0]=0,$   
 $Q[0, 0, 3, 0]=0,$   
 $Q1[3, 0, 0, 1]=0, Q2[3, 0, 0, 1]=0,$   
 $Q1[2, 1, 0, 1]=0, Q2[2, 1, 0, 1]=0,$   
 $Q1[1, 0, 1, 1]=0, Q2[1, 0, 1, 1]=0,$   
 $Q[0, 0, 0, 2]=0$

}:

> sol:=solve(EQ):

> sol[3];

$$\{F02 = 0, h41 = -\frac{12785860472748136949}{147484825378985652098}r^6, H41 = 0, h002 = 0, h6 = 0, H6 = 0,$$

$$G5 = 0, g5 = 0, r = r, f101 = 0, F21 = \frac{40232017790685}{161596568956226}r^4,$$

$$H22 = \frac{317151709775240372613}{147484825378985652098}r^6, f02 = \frac{40232017790685}{323193137912452}r^4, a = 0,$$

$$D2 = \frac{158768815275}{6663776039432}r^3, H111 = -\frac{47713089081230852517}{147484825378985652098}r^6,$$

$$f4 = -\frac{65520714687687}{323193137912452}r^4, h301 = -\frac{47713089081230852517}{147484825378985652098}r^6,$$

$$G12 = \frac{8081586852126501}{15674867188753922}r^5, F101 = -\frac{56324824906959}{323193137912452}r^4, h111 = 0, G201 = 0,$$

$$g011 = 0, F4 = 0, G31 = 0, f21 = 0, h22 = 0, E1 = 0, D1 = 0, E2 = \frac{95261289165}{3331888019716}r^3,$$

$$m = \frac{278}{97}, g201 = -\frac{1456399044022797}{15674867188753922}r^5, H03 = 0,$$

$$h03 = -\frac{4365903576060339446}{73742412689492826049} r^6, g31 = -\frac{390276868006109}{15674867188753922} r^5, H301 = 0,$$

$$G011 = -\frac{13107591396205173}{31349734377507844} r^5, g12 = 0, H002 = -\frac{715696336218462787755}{589939301515942608392} r^6\}$$

```
> sol[4];
{ G201 = -2 r^5, g011 = r^5, h6 = 0, H6 = 0, h002 = 0, G5 = 0, g5 = 0, r = r, f101 = 0, a = r^2,
  F4 = -1/2 r^4, G31 = 6 r^5, m = -1, f02 = 1/2 r^4, F21 = r^4, D2 = 0, H111 = 8 r^6, H22 = 32 r^6,
  h301 = 8 r^6, F101 = 0, f4 = -1/2 r^4, h111 = 24 r^6, D1 = 0, E1 = 0, f21 = -r^4, G12 = 6 r^5,
  F02 = 1/2 r^4, h41 = -24 r^6, E2 = 0, h22 = -48 r^6, H301 = -8 r^6, g201 = r^5, h03 = -8 r^6,
  H002 = -4 r^6, H41 = 24 r^6, H03 = 8 r^6, g31 = -2 r^5, g12 = 0, G011 = -r^5 }
```

>

So, it is not the cubic for n=0.

## Type II for non-zero gamma

First step: (3,4)-normalization ----- (2,3,4) jet

```
> restart: readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=x*(1+y)*ln(1+y)+n*((1+y)^2-1):
T:=(1+y)*ln(1+y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],9):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=2*S2/n:
T3:=32*n*(T2-S3/4):
```

**Result check.**

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz))))/2;
```

```

T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z))));
      S1,0,0,0:=0
      S1,1,0,0:=1
      S2,0,0,0:=0
      xxxxxxxxxxxxxxxxxxxxxxxxxxxx
      T1,0,0,0:=0
      T1,1,0,0:=0
      T2,0,0,0:=0
      T2,1,0,0:=2 I n - 1
      T3,0,0,0:=0

```

**Introduce the notations.**

```

> k:=2*I*n-1: ck:=-2*I*n-1: m:=1+4*n^2:

```

**Introduce new variables**

```

> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):

```

**Write a mapping**

```

> f:=
      r*ck*z+
      (F2+I*f2)*z^2+(F01+I*f01)*w2:

      cf:=
      r*k*cz+
      (F2-I*f2)*cz^2+(F01-I*f01)*cw2:

      g:=
      r^2*m*w2+
      (G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:

      cg:=
      r^2*m*cw2+
      (G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:

      h:=
      r^3*m^2*w3+
      (H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+

```



(H101+I\*h101)\*z\*w3:

```
ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (3,4)-jet.**

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):
```

**Write the equations and the variables.**

```
> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,
```

```
Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
```

```

Q1 [3, 1, 0, 0]=0, Q2 [3, 1, 0, 0]=0,
  Q [2, 2, 0, 0]=0,
Q1 [2, 0, 1, 0]=0, Q2 [2, 0, 1, 0]=0,
  Q [1, 1, 1, 0]=0,
Q1 [1, 0, 0, 1]=0, Q2 [1, 0, 0, 1]=0
  } :

```

**17 equations Solve the system.**

```
> A:=0:s0:=solve(EQ) [2];
```

```
s0 := {H21 = 8/3 r^4 n^3 - 16 r^2 f01 n^2 + 1/3 r^4 n - 2 r^2 f01 + 16/3 r^4 n^5 - 32 r^2 n^4 f01, r = r, n = n,
```

```
F01 = 1/3 r^2 n^2 - 2 n f01 + 1/12 r^2, F2 = 1/3 n r^2 + f01, H101 = 0, h101 = 0, g001 = 0,
```

```
h4 = 0, H4 = 0, g3 = 0, G3 = 0,
```

```
G001 = - 1/8 (r (r^2 + 16 r^2 n^4 - 32 f01 n^3 + 8 r^2 n^2 - 8 n f01) / n,
```

```
H02 = - 4/3 n^2 r^4 - 1/6 r^4 - 8/3 r^4 n^4, g11 = - 2/3 r^3 n^2 - 1/6 r^3,
```

```
G11 = 4/3 n^3 r^3 - 8 n^2 r f01 + 1/3 n r^3 - 2 r f01, f2 = 1/3 r^2 n^2 - 1/12 r^2 + 2 n f01, f01 = f01,
```

```
h02 = - 32 r^2 n^4 f01 - 16 r^2 f01 n^2 - 2 r^2 f01, h21 = - 4/3 n^2 r^4 - 1/6 r^4 - 8/3 r^4 n^4}
```

>

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```
> restart:readlib(mttaylor):
```

```
A:=0: x:=(z+cz)/2:y:=(z-cz)/(2*I):
```

```
S:=x*(1+y)*ln(1+y)+n*((1+y)^2-1):
```

```
T:=(1+y)*ln(1+y):
```

```
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],9):
```

```
S2:=int(diff(int(diff(S1,z),z),cz),cz):
```

```
T2:=int(diff(int(diff(T1,z),z),cz),cz):
```

```
S3:=2*S2/n:
```

```
T3:=32*n*(T2-S3/4):
```

```
k:=2*I*n-1: ck:=-2*I*n-1: m:=1+4*n^2:
```

**Fix the results of the previous - (3,4) - step.**

```
> f01:=a:
```

```
H21:= 8/3*r^4*n^3-16*r^2*f01*n^2+1/3*r^4*n-2*r^2*f01+16/3*r^4*n^5-
  32*r^2*n^4*f01:
```

```
F01:= 1/3*r^2*n^2-2*n*f01+1/12*r^2:
```

```
F2:= 1/3*n*r^2+f01: H101:= 0: h101:= 0: g001:= 0: h4:= 0: H4:= 0:
g3:= 0: G3:= 0: G001:= -1/8*r*(r^2+16*r^2*n^4-32*f01*n^3+8*r^2*n^2-
  8*n*f01)/n: H02:= -4/3*n^2*r^4-1/6*r^4-8/3*r^4*n^4: g11:=
```

```

-2/3*r^3*n^2-1/6*r^3: G11:= 4/3*n^3*r^3-8*n^2*r*f01+1/3*n*r^3-
2*r*f01: f2:= 1/3*r^2*n^2-1/12*r^2+2*n*f01:
h02:= -32*r^2*n^4*f01-16*r^2*f01*n^2-2*r^2*f01: h21:= -4/3*n^2*r^4-
1/6*r^4-8/3*r^4*n^4:

```

**Introduce new variables**

```

> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):

```

**Write a mapping**

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

**Write the relations for the mapping.**

```
> e2:=subs ([z=f, cz=cf], 2*S3)+I*(g-cg) :  
e3:=subs ([z=f, cz=cf], 2*T3)+I*(h-ch) :
```

**Write the coefficients of the (4,5)-jet.**

```
> P[4,0,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z, z))) :  
P[3,1,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z, cz))) :  
P[2,2,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, cz, cz))) :  
P[2,0,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, u2))) :  
P[1,1,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, cz, u2))) :  
P[1,0,0,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, u3))) :
```

```
P1[4,0,0,0]:=subs (I=0, P[4,0,0,0]) : P2[4,0,0,0]:=expand((P[4,0,0,0]-  
P1[4,0,0,0])*I) :  
P1[3,1,0,0]:=subs (I=0, P[3,1,0,0]) : P2[3,1,0,0]:=expand((P[3,1,0,0]-  
P1[3,1,0,0])*I) :  
P1[2,0,1,0]:=subs (I=0, P[2,0,1,0]) : P2[2,0,1,0]:=expand((P[2,0,1,0]-  
P1[2,0,1,0])*I) :  
P1[1,0,0,1]:=subs (I=0, P[1,0,0,1]) : P2[1,0,0,1]:=expand((P[1,0,0,1]-  
P1[1,0,0,1])*I) :
```

```
Q[5,0,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, z, z))) :  
Q[4,1,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, z, cz))) :  
Q[3,2,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, cz, cz))) :  
:  
Q[3,0,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, u2))) :  
Q[2,1,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, cz, u2))) :  
Q[1,0,2,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, u2, u2))) :  
Q[2,0,0,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, u3))) :  
Q[1,1,0,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, cz, u3))) :  
Q[0,0,1,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, u2, u3))) :
```

```
Q1[5,0,0,0]:=subs (I=0, Q[5,0,0,0]) : Q2[5,0,0,0]:=expand((Q[5,0,0,0]-  
Q1[5,0,0,0])*I) :  
Q1[4,1,0,0]:=subs (I=0, Q[4,1,0,0]) : Q2[4,1,0,0]:=expand((Q[4,1,0,0]-  
Q1[4,1,0,0])*I) :  
Q1[3,2,0,0]:=subs (I=0, Q[3,2,0,0]) : Q2[3,2,0,0]:=expand((Q[3,2,0,0]-  
Q1[3,2,0,0])*I) :  
Q1[3,0,1,0]:=subs (I=0, Q[3,0,1,0]) : Q2[3,0,1,0]:=expand((Q[3,0,1,0]-  
Q1[3,0,1,0])*I) :  
Q1[2,1,1,0]:=subs (I=0, Q[2,1,1,0]) : Q2[2,1,1,0]:=expand((Q[2,1,1,0]-  
Q1[2,1,1,0])*I) :  
Q1[1,0,2,0]:=subs (I=0, Q[1,0,2,0]) : Q2[1,0,2,0]:=expand((Q[1,0,2,0]-  
Q1[1,0,2,0])*I) :  
Q1[2,0,0,1]:=subs (I=0, Q[2,0,0,1]) : Q2[2,0,0,1]:=expand((Q[2,0,0,1]-  
Q1[2,0,0,1])*I) :
```

**Write the equations and the variables.**

```
> X:={  
g4, H5, h5, G4, h011, F11, f11, g02, G02, F3, f001, F001, f3, C1, C2, H31, H12, h12,
```

```

H011,g101,H201,G101,
h201,g21,G21,h31}:
> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Number of equations - 26.**

```
> sol:=solve(EQ,X):
```

**The (3,4,5)-jet, as well as C1 and C2 are defined modulo r and f01=a. The coefficient B is free, and we can put B=0.**

```
> B:=0:solve(EQ,X);
```

$$\{C1 = \frac{1}{72} \frac{4n^2 r^4 + 432 a^2 + 3 r^4}{r^2}, f001 = \frac{1}{288} \frac{4n^2 r^4 + 144 a^2 + r^4}{r}, C2 = -\frac{1}{54} n r^2,$$

$$F3 = -\frac{1}{144} \frac{-12 n^2 r^4 + 96 n r^2 a + r^4 + 144 a^2}{r},$$

$$H12 = -\frac{1}{18} r^5 n + \frac{8}{3} n^2 r^3 a - \frac{4}{9} r^5 n^3 - \frac{8}{9} r^5 n^5 + \frac{1}{3} r^3 a + \frac{16}{3} r^3 n^4 a, h12 = 2 r a^2 + \frac{1}{24} r^5$$

$$+ 32 r n^4 a^2 - \frac{2}{3} n r^3 a - \frac{16}{3} r^3 n^3 a + 16 r n^2 a^2 - \frac{32}{3} r^3 n^5 a + \frac{7}{18} n^2 r^5 + \frac{10}{9} r^5 n^4$$

$$+ \frac{8}{9} r^5 n^6,$$

$$G21 = \frac{1}{72} \frac{-24 r^4 n^3 + 72 r^2 a n^2 - 5 r^4 n + 18 r^2 a - 16 r^4 n^5 + 576 n^3 a^2 + 144 a^2 n}{n},$$

$$f11 = -\frac{1}{27}r(-nr^2 + 18a)n,$$

$$F001 = -\frac{1}{144}\frac{-4r^4n^3 - 48r^2an^2 - r^4n + 144a^2n - 12r^2a}{r}, F11 = \frac{1}{54}nr^3 - \frac{1}{3}ra,$$

$$f3 = -\frac{1}{72}\frac{-4r^4n^3 + 3r^4n + 48r^2an^2 + 144a^2n - 12r^2a}{r},$$

$$g21 = \frac{1}{144}\frac{r^2(-16r^2n^4 + 288an^3 + 72na + 3r^2 + 8r^2n^2)}{n},$$

$$h201 = -\frac{2}{9}r^5n^3 - \frac{4}{3}n^2r^3a - \frac{1}{36}r^5n - \frac{1}{6}r^3a - \frac{4}{9}r^5n^5 - \frac{8}{3}r^3n^4a, H5 = 0, h5 = 0,$$

$$G4 = 0, g4 = 0, h011 = 0,$$

$$h31 = \frac{2}{3}r^3a - \frac{16}{9}r^5n^5 - \frac{8}{9}r^5n^3 + \frac{16}{3}n^2r^3a + \frac{32}{3}r^3n^4a - \frac{1}{9}r^5n,$$

$$g02 = -\frac{1}{36}\frac{-96r^2n^4a + 576n^3a^2 + 4r^4n^3 - 60r^2an^2 + r^4n - 9r^2a + 144a^2n}{n},$$

$$H201 = -\frac{1}{36}n^2r^5 - ra^2 - \frac{1}{144}r^5 + \frac{4}{9}r^5n^6 + \frac{16}{3}r^3n^5a + \frac{1}{9}r^5n^4 - 16rn^4a^2$$

$$+ \frac{8}{3}r^3n^3a - 8rn^2a^2 + \frac{1}{3}nr^3a,$$

$$G02 = -\frac{1}{432}\frac{r^2(-112r^2n^4 + 288an^3 - 64r^2n^2 + 72na - 9r^2)}{n},$$

$$H011 = -\frac{4}{9}r^5n^3 - \frac{8}{3}n^2r^3a - \frac{1}{18}r^5n - \frac{1}{3}r^3a - \frac{8}{9}r^5n^5 - \frac{16}{3}r^3n^4a,$$

$$G101 = \frac{1}{9}r^4n^4 + \frac{4}{3}r^2an^3 - 4n^2a^2 + \frac{1}{3}nr^2a - a^2 - \frac{1}{144}r^4,$$

$$g101 = -\frac{1}{9}r^4n^3 - \frac{2}{3}r^2an^2 - \frac{1}{36}r^4n - \frac{1}{6}r^2a, H31 = \frac{16}{9}r^5n^6 + 64rn^4a^2 - \frac{64}{3}r^3n^5a$$

$$+ \frac{4}{9}r^5n^4 - \frac{1}{9}n^2r^5 - \frac{1}{36}r^5 + 32rn^2a^2 + 4ra^2 - \frac{4}{3}nr^3a - \frac{32}{3}r^3n^3a\}$$

$$> \text{CC1} := 1/72*(4*n^2*r^4+432*a^2+3*r^4)/(r^2);$$

$$CC2 := -1/54 * n * r^2;$$

$$CCI := \frac{1}{72} \frac{4 n^2 r^4 + 432 a^2 + 3 r^4}{r^2}$$

$$CC2 := -\frac{1}{54} n r^2$$

**So if gamma is non-zero - it is not the cubic.**

**Type II for gamma=0.**

**First step: (3,4)-normalization ----- (2,3,4)**

**jet**

```
> restart:readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
A:=0:
T:=x*(1+y)*ln(1+y):
S:=(1+y)*ln(1+y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=4*S2:
T3:=16*T2:
```

**Result check.**

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 1$$

$$T_{3,0,0,0} := 0$$

**Introduce the notations.**

> k:=1: ck:=1: m:=1:

**Introduce new variables**

> w2:=u2+I\*z\*cz: cw2:=u2-I\*z\*cz:  
w3:=u3+I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):  
cw3:=u3-I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):

**Write a mapping**

> f:=  
r\*ck\*z+  
(F2+I\*f2)\*z^2+(F01+I\*f01)\*w2:  
  
cf:=  
r\*k\*cz+  
(F2-I\*f2)\*cz^2+(F01-I\*f01)\*cw2:

g:=  
r^2\*m\*w2+  
(G3+I\*g3)\*z^3+(G11+I\*g11)\*z\*w2+(G001+I\*g001)\*w3:

cg:=  
r^2\*m\*cw2+  
(G3-I\*g3)\*cz^3+(G11-I\*g11)\*cz\*cw2+(G001-I\*g001)\*cw3:

h:=  
r^3\*m^2\*w3+  
(H4+I\*h4)\*z^4+(H21+I\*h21)\*z^2\*w2+(H02+I\*h02)\*w2^2+  
(H101+I\*h101)\*z\*w3:

ch:=  
r^3\*m^2\*cw3+  
(H4-I\*h4)\*cz^4+(H21-I\*h21)\*cz^2\*cw2+(H02-I\*h02)\*cw2^2+(H101-I\*h101)\*cz\*cw3:

**Write the relations for the mapping.**

> e2:=subs([z=f, cz=cf], 2\*S3)+I\*(g-cg):  
e3:=subs([z=f, cz=cf], 2\*T3)+I\*(h-ch):

**Write the coefficients of the (3,4)-jet.**

> P[3,0,0,0]:=expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z))):  
P[2,1,0,0]:=expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, cz))):



```

P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):

P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):

Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**17 equations Solve the system.**

```

> s0:=solve(EQ)[2];

```

$$s0 := \{F01 = \frac{1}{12}r^2, f2 = -\frac{1}{12}r^2, F2 = f01, h02 = 2r^2f01, G3 = 0, g3 = 0, H4 = 0, h4 = 0,$$

$$g001 = 0, H101 = 0, h101 = 0, h21 = \frac{1}{6}r^4, G11 = 2rf01, G001 = -rf01, g11 = \frac{1}{6}r^3,$$

$$H02 = \frac{1}{6}r^4, r = r, f01 = f01, H21 = 2r^2 f01 \}$$

>

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```
> restart:readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
A:=0:B:=0:
T:=x*(1+y)*ln(1+y):
S:=(1+y)*ln(1+y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=4*S2:
T3:=16*T2:
k:=1: ck:=1: m:=1:
```

**Fix the results of the previous - (3,4) - step.**

```
> f01:=a: h02:= 2*r^2*f01: h21:= 1/6*r^4: F01:= 1/12*r^2: F2:= f01:
g3:= 0: H4:= 0: h4:= 0: g001:= 0: H101:= 0: h101:= 0: G3:= 0: g11:=
1/6*r^3: G001:= -r*f01: H02:= 1/6*r^4: G11:= 2*r*f01: H21:=
2*r^2*f01: f2:= -1/12*r^2:
```

**Introduce new variables**

```
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
```

**Write a mapping**

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

cg:=
r^2*m*cw2+
```

$$(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-I*g101)*cz*cw3:$$

$$h:=r^3*m^2*w3+(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+(H101+I*h101)*z*w3+(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:$$

$$ch:=r^3*m^2*cw3+(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-I*h101)*cz*cw3+(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:$$

**Write the relations for the mapping.**

$$\begin{aligned} > e2:=\text{subs}([z=f, cz=cf], 2*S3)+I*(g-cg): \\ e3:=\text{subs}([z=f, cz=cf], 2*T3)+I*(h-ch): \end{aligned}$$

**Write the coefficients of the (4,5)-jet.**

$$\begin{aligned} > P[4,0,0,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, z, z))): \\ P[3,1,0,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, z, cz))): \\ P[2,2,0,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, cz, cz))): \\ P[2,0,1,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, u2))): \\ P[1,1,1,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, cz, u2))): \\ P[1,0,0,1] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, u3))): \\ \\ P1[4,0,0,0] &:=\text{subs}(I=0, P[4,0,0,0]): P2[4,0,0,0] :=\text{expand}((P[4,0,0,0]-P1[4,0,0,0])*I): \\ P1[3,1,0,0] &:=\text{subs}(I=0, P[3,1,0,0]): P2[3,1,0,0] :=\text{expand}((P[3,1,0,0]-P1[3,1,0,0])*I): \\ P1[2,0,1,0] &:=\text{subs}(I=0, P[2,0,1,0]): P2[2,0,1,0] :=\text{expand}((P[2,0,1,0]-P1[2,0,1,0])*I): \\ P1[1,0,0,1] &:=\text{subs}(I=0, P[1,0,0,1]): P2[1,0,0,1] :=\text{expand}((P[1,0,0,1]-P1[1,0,0,1])*I): \\ \\ Q[5,0,0,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, z, z))): \\ Q[4,1,0,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, z, cz))): \\ Q[3,2,0,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, cz, cz))): \\ &: \\ Q[3,0,1,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, u2))): \\ Q[2,1,1,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, cz, u2))): \\ Q[1,0,2,0] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, u2, u2))): \\ Q[2,0,0,1] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, u3))): \\ Q[1,1,0,1] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, cz, u3))): \\ Q[0,0,1,1] &:=\text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, u2, u3))): \end{aligned}$$

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Number of equations - 26.**

```

> sol:=solve(EQ):
> sol[2][20];

$$CI = \frac{1}{24} \frac{r^4 + 144 a^2}{r^2}$$


```

>

**So if gamma is zero - then it is not a cubic as well.**

**Type II for gamma=0.**

# First step: (3,4)-normalization ----- (2,3,4) jet

```
> restart:readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
A:=0:
T:=x*(1+y)*ln(1+y):
S:=(1+y)*ln(1+y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=4*S2:
T3:=16*T2:
```

## Result check.

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))):
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))):
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))):
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))):
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))):
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))):
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz))))/2:
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))):
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 1$$

$$T_{3,0,0,0} := 0$$

## Introduce the notations.

```
> k:=1: ck:=1: m:=1:
```

## Introduce new variables

```
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):
```

### Write a mapping

```
> f:=
  r*ck*z+
  (F2+I*f2)*z^2+(F01+I*f01)*w2:

  cf:=
  r*k*cz+
  (F2-I*f2)*cz^2+(F01-I*f01)*cw2:

  g:=
  r^2*m*w2+
  (G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:

  cg:=
  r^2*m*cw2+
  (G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:

  h:=
  r^3*m^2*w3+
  (H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
  (H101+I*h101)*z*w3:

  ch:=
  r^3*m^2*cw3+
  (H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
  I*h101)*cz*cw3:
```

### Write the relations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
  e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

### Write the coefficients of the (3,4)-jet.

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
  P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
  P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
  P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):

  P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
  P1[3,0,0,0])*I):
  P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
  P1[2,1,0,0])*I):
  P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
  P1[1,0,1,0])*I):
  P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
  P1[0,0,0,1])*I):

  Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
```

```

Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**17 equations Solve the system.**

```

> s0:=solve(EQ)[2];
s0:={F01=1/12*r^2,f2=-1/12*r^2,F2=f01,h02=2*r^2*f01,G3=0,g3=0,H4=0,h4=0,
g001=0,H101=0,h101=0,h21=1/6*r^4,G11=2*r*f01,G001=-r*f01,g11=1/6*r^3,
H02=1/6*r^4,r=r,f01=f01,H21=2*r^2*f01}

```

>

**Second Step: (4,5)-normalization ----- (3,4,5) jet**

```

> restart:readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
A:=0:B:=0:
T:=x*(1+y)*ln(1+y):
S:=(1+y)*ln(1+y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):

```

```
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=4*S2:
T3:=16*T2:
k:=1: ck:=1: m:=1:
```

**Fix the results of the previous - (3,4) - step.**

```
> f01:=a: h02:= 2*r^2*f01: h21:= 1/6*r^4: F01:= 1/12*r^2: F2:= f01:
g3:= 0: H4:= 0: h4:= 0: g001:= 0: H101:= 0: h101:= 0: G3:= 0: g11:=
1/6*r^3: G001:= -r*f01: H02:= 1/6*r^4: G11:= 2*r*f01: H21:=
2*r^2*f01: f2:= -1/12*r^2:
```

**Introduce new variables**

```
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
```

**Write a mapping**

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:
```

```
g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:
```

```
cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:
```

```
h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:
```

```
ch:=
```



$$\begin{aligned}
& r^3 m^2 c w^3 + \\
& (H4 - I h4) * c z^4 + (H21 - I h21) * c z^2 * c w^2 + (H02 - I h02) * c w^2^2 + (H101 - \\
& \quad I h101) * c z * c w^3 + \\
& (H5 - I h5) * c z^5 + (H31 - I h31) * c z^3 * c w^2 + (H12 - I h12) * c z * c w^2^2 + (H201 - \\
& \quad I h201) * c z^2 * c w^3 + (H011 - I h011) * c w^2 * c w^3 :
\end{aligned}$$

**Write the relations for the mapping.**

$$\begin{aligned}
> e2 &:= \text{subs}([z=f, cz=cf], 2*S3) + I*(g-cg) : \\
e3 &:= \text{subs}([z=f, cz=cf], 2*T3) + I*(h-ch) :
\end{aligned}$$

**Write the coefficients of the (4,5)-jet.**

$$\begin{aligned}
> P[4,0,0,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, z, z))) : \\
P[3,1,0,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, z, cz))) : \\
P[2,2,0,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, cz, cz))) : \\
P[2,0,1,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, z, u2))) : \\
P[1,1,1,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, cz, u2))) : \\
P[1,0,0,1] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e2, z, u3))) : \\
\\
P1[4,0,0,0] &:= \text{subs}(I=0, P[4,0,0,0]) : P2[4,0,0,0] := \text{expand}((P[4,0,0,0] - \\
& \quad P1[4,0,0,0]) * I) : \\
P1[3,1,0,0] &:= \text{subs}(I=0, P[3,1,0,0]) : P2[3,1,0,0] := \text{expand}((P[3,1,0,0] - \\
& \quad P1[3,1,0,0]) * I) : \\
P1[2,0,1,0] &:= \text{subs}(I=0, P[2,0,1,0]) : P2[2,0,1,0] := \text{expand}((P[2,0,1,0] - \\
& \quad P1[2,0,1,0]) * I) : \\
P1[1,0,0,1] &:= \text{subs}(I=0, P[1,0,0,1]) : P2[1,0,0,1] := \text{expand}((P[1,0,0,1] - \\
& \quad P1[1,0,0,1]) * I) : \\
\\
Q[5,0,0,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, z, z))) : \\
Q[4,1,0,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, z, cz))) : \\
Q[3,2,0,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, cz, cz))) : \\
& \quad : \\
Q[3,0,1,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, z, u2))) : \\
Q[2,1,1,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, cz, u2))) : \\
Q[1,0,2,0] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, u2, u2))) : \\
Q[2,0,0,1] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, z, u3))) : \\
Q[1,1,0,1] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, z, cz, u3))) : \\
Q[0,0,1,1] &:= \text{expand}(\text{subs}([z=0, cz=0, u2=0, u3=0], \text{diff}(e3, u2, u3))) : \\
\\
Q1[5,0,0,0] &:= \text{subs}(I=0, Q[5,0,0,0]) : Q2[5,0,0,0] := \text{expand}((Q[5,0,0,0] - \\
& \quad Q1[5,0,0,0]) * I) : \\
Q1[4,1,0,0] &:= \text{subs}(I=0, Q[4,1,0,0]) : Q2[4,1,0,0] := \text{expand}((Q[4,1,0,0] - \\
& \quad Q1[4,1,0,0]) * I) : \\
Q1[3,2,0,0] &:= \text{subs}(I=0, Q[3,2,0,0]) : Q2[3,2,0,0] := \text{expand}((Q[3,2,0,0] - \\
& \quad Q1[3,2,0,0]) * I) : \\
Q1[3,0,1,0] &:= \text{subs}(I=0, Q[3,0,1,0]) : Q2[3,0,1,0] := \text{expand}((Q[3,0,1,0] - \\
& \quad Q1[3,0,1,0]) * I) : \\
Q1[2,1,1,0] &:= \text{subs}(I=0, Q[2,1,1,0]) : Q2[2,1,1,0] := \text{expand}((Q[2,1,1,0] - \\
& \quad Q1[2,1,1,0]) * I) : \\
Q1[1,0,2,0] &:= \text{subs}(I=0, Q[1,0,2,0]) : Q2[1,0,2,0] := \text{expand}((Q[1,0,2,0] - \\
& \quad Q1[1,0,2,0]) * I) :
\end{aligned}$$

```
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):
```

**Write the equations and the variables.**

```
> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:
```

**Number of equations - 26.**

```
> sol:=solve(EQ):
> sol[2][20];

$$CI = \frac{1}{24} \frac{r^4 + 144 a^2}{r^2}$$

```

>

**So if gamma is zero - then it is not a cubic as well.**

## Type IIIa

First step: (3,4)-normalization -----

(2,3,4) - jet

```
> restart:readlib(mtaylor):
A:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
T:=x*sqrt(1-y^2)+n*arcsin(y):
S:=sqrt(1-y^2):
S1:=mtaylor(-4*S,[z,cz],8): T1:=mtaylor(-16*T,[z,cz],9):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
```

T3:=T2:

**Result check.**

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z))));  
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz))));  
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z))));  
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');  
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z))));  
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz))));  
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z))));  
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz))))/2;  
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z))));
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := In + 1$$

$$T_{3,0,0,0} := 0$$

**Introduce the notations.**

```
> k:=1+I*n: ck:=1-I*n: m:=1+n^2:
```

**Introduce new variables**

```
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:  
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):  
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):
```

**Write a mapping**

```
> f:=  
r*ck*z+  
(F2+I*f2)*z^2+(F01+I*f01)*w2:
```

```
cf:=  
r*k*cz+  
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:
```

```
g:=  
r^2*m*w2+  
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

**Write the relations for a mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (3,4)-jet.**

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
```

```
Q1[1,0,0,1])*I):
```

**Write the equations and the variables.**

```
> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**17 equations      Solve the sustem.**

```
> s:=solve(EQ)[2];
s:={f2=-n*f01,h101=0,H4=0,h4=0,g3=0,G3=0,g001=0,H101=0,H02=0,
n=n,F01=n*f01,f01=f01,F2=f01,h02=2*r^2*f01+2*r^2*n^4*f01+4*r^2*n^2*f01,
h21=0,H21=2*r^2*f01+2*r^2*n^4*f01+4*r^2*n^2*f01,G001=-r*n^2*f01-r*f01,r=r,
g11=0,G11=2*r*n^2*f01+2*r*f01}

```

```
>
```

**Seconf step: (4,5)-normalization ----- (3,4,5) jet**

```
> restart:readlib(mttaylor):
A:=0:B:=0:
x:=(z+cz)/2:y:=(z-cz)/(2*I):
T:=x*sqrt(1-y^2)+n*arcsin(y):
S:=sqrt(1-y^2):
S1:=mtaylor(-4*S,[z,cz],8): T1:=mtaylor(-16*T,[z,cz],9):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=T2:
k:=1+I*n: ck:=1-I*n: m:=1+n^2:
Write the result of the previous - (3,4) - step.
> f01:=a:F2:= f01: h21:= 0: h02:=
2*r^2*n^4*f01+4*r^2*n^2*f01+2*r^2*f01: H4:= 0: h4:= 0: h101:= 0:
g3:= 0: g001:= 0:
H101:= 0: G3:= 0: H02:= 0: H21:=
2*r^2*n^4*f01+4*r^2*n^2*f01+2*r^2*f01: G001:= -r*n^2*f01-r*f01:
g11:= 0:
G11:= 2*r*n^2*f01+2*r*f01: F01:= n*f01: f2:= -n*f01:

```

## Introduce new variables

```
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):  
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-I  
I*C2)*cz^3*z^2):  
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-I  
I*C2)*cz^3*z^2):
```

## Write a mapping

```
> f:=  
r*ck*z+  
(F2+I*f2)*z^2+(F01+I*f01)*w2+  
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:
```

```
cf:=  
r*k*cz+  
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+  
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:
```

```
g:=  
r^2*m*w2+  
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+  
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+  
(G101+I*g101)*z*w3:
```

```
cg:=  
r^2*m*cw2+  
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+  
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-I  
I*g101)*cz*cw3:
```

```
h:=  
r^3*m^2*w3+  
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+  
(H101+I*h101)*z*w3+  
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+  
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:
```

```
ch:=  
r^3*m^2*cw3+  
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-I  
I*h101)*cz*cw3+  
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-I  
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:
```

## Write the reations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):  
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

### Write the coefficients of the (4,5)-jet.

```
> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):
```

```
P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):
```

```
Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):
```

```
Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):
```

### Write the equations and te variables.

```
> X:={
g4,H5,h5,G4,h011,F11,f11,g02,G02,F3,f001,F001,f3,C1,C2,H31,H12,h12,
H011,g101,H201,G101,
h201,g21,G21,h31}:
> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
```

P1[3,1,0,0]=0,P2[3,1,0,0]=0,  
P[2,2,0,0]=0,  
P1[2,0,1,0]=0,P2[2,0,1,0]=0,  
P[1,1,1,0]=0,  
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,  
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,  
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,  
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,  
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,  
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,  
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,  
Q[1,1,0,1]=0,  
Q[0,0,1,1]=0  
}:

## 26 equations.

> sol:=solve(EQ):

> sol[4];

{h31 = 0, h011 = 0, H12 = 0, H5 = 0, h5 = 0, G4 = 0, g4 = 0, g21 = 0,

$$H31 = 4 r n^4 a^2 + 8 r n^2 a^2 + 4 r a^2, G02 = \frac{1}{12} r^4 n + \frac{1}{12} r^4 n^3, G21 = 2 a^2 + 2 n^2 a^2,$$

$$G101 = \frac{1}{16} r^4 n^4 - n^2 a^2 - a^2 - \frac{1}{16} r^4, h12 = 2 r n^4 a^2 + 4 r n^2 a^2 + 2 r a^2,$$

$$g101 = \frac{1}{8} r^4 n^3 + \frac{1}{8} r^4 n, n = n, a = a, r = r,$$

$$H201 = \frac{1}{16} r^5 n^6 - r n^4 a^2 + \frac{1}{16} r^5 n^4 - 2 r n^2 a^2 - \frac{1}{16} r^5 n^2 - r a^2 - \frac{1}{16} r^5,$$

$$f001 = -\frac{1}{32} \frac{r^4 n^2 + 16 a^2 + r^4}{r}, C1 = \frac{1}{8} \frac{48 a^2 + r^4 n^2 + 3 r^4}{r^2}, F11 = \frac{1}{12} r^3 n,$$

$$f3 = \frac{1}{16} \frac{n(r^4 n^2 - 3 r^4 - 16 a^2)}{r}, F001 = \frac{1}{32} \frac{n(r^4 n^2 - 16 a^2 + r^4)}{r}, C2 = \frac{1}{12} r^2 n,$$

$$f11 = -\frac{1}{12} n^2 r^3, F3 = -\frac{1}{16} \frac{3 r^4 n^2 - 16 a^2 - r^4}{r}, h201 = \frac{1}{8} r^5 n^5 + \frac{1}{4} r^5 n^3 + \frac{1}{8} r^5 n,$$

$$g02 = -\frac{3}{8} r^4 n^2 - 4 a^2 - \frac{5}{16} r^4 - \frac{1}{16} r^4 n^4 - 4 n^2 a^2, H011 = \frac{1}{4} r^5 n^5 + \frac{1}{2} r^5 n^3 + \frac{1}{4} r^5 n\}$$

> CC1:= 1/8\*(48\*a^2+r^4\*n^2+3\*r^4)/(r^2);



$$CCI := \frac{1}{8} \frac{48 a^2 + r^4 n^2 + 3 r^4}{r^2}$$

**Then we see that it is not a cubic for all n**

## **Type IIIb.**

Here we put  $q:=1/q$

### **Preparations.**

**We solve the implicit equation  $F=2*\arctg(P/(1+y))-q*\ln((1+y)^2+P^2)=0$  with respect to P with the condition  $P(0)=0$ .**

```
restart:readlib(mttaylor):
a[1]:= q:
a[2]:= 1/2*q+1/2*q^3:
a[3]:= 1/3*q^3+1/2*q^5-1/6*q:
a[4]:= 1/3*q^5+5/8*q^7-5/24*q^3+1/12*q:
a[5]:= -1/20*q-13/40*q^5+3/8*q^7+1/8*q^3+7/8*q^9:
a[6]:= 161/720*q^5-389/720*q^7+7/16*q^9+21/16*q^11-7/90*q^3+1/30*q:
a[7]:= -1/42*q+1/20*q^3-11/80*q^5+179/420*q^7-
37/40*q^9+1/2*q^11+33/16*q^13:
a[8]:= -331/10080*q^3+73/960*q^5-3547/13440*q^7+4213/5040*q^9-
103/64*q^11+33/64*q^13+1/56*q+429/128*q^15:
>
> P:=normal(sum('a[k]*y^k', 'k'=1..8));
```

$$\begin{aligned}
P := & \frac{1}{2}y^2q + a_8y^8 + qy + \frac{1}{2}y^2q^3 + \frac{1}{3}y^3q^3 + \frac{1}{2}y^3q^5 - \frac{1}{6}y^3q + \frac{1}{3}y^4q^5 + \frac{5}{8}y^4q^7 - \frac{5}{24}y^4q^3 \\
& + \frac{1}{12}y^4q - \frac{1}{20}y^5q - \frac{13}{40}y^5q^5 + \frac{3}{8}y^5q^7 + \frac{1}{8}y^5q^3 + \frac{7}{8}y^5q^9 + \frac{161}{720}y^6q^5 - \frac{389}{720}y^6q^7 \\
& + \frac{7}{16}y^6q^9 + \frac{21}{16}y^6q^{11} - \frac{7}{90}y^6q^3 + \frac{1}{30}y^6q - \frac{1}{42}y^7q + \frac{1}{20}y^7q^3 - \frac{11}{80}y^7q^5 + \frac{179}{420}y^7q^7 \\
& - \frac{37}{40}y^7q^9 + \frac{1}{2}y^7q^{11} + \frac{33}{16}y^7q^{13}
\end{aligned}$$

```

> F:=mtaylor(2*arctan(P/(1+y))-q*ln((1+y)^2+P^2),y,9):
  > b[1]:=solve(coeff(F,y,1)=0,a[1]);
      b1:=q

  > b[2]:=solve(coeff(F,y,2)=0,a[2]);
      b2:= $\frac{1}{2}q + \frac{1}{2}q^3$ 

  > b[3]:=solve(coeff(F,y,3)=0,a[3]);
      b3:= $\frac{1}{3}q^3 + \frac{1}{2}q^5 - \frac{1}{6}q$ 

  > b[4]:=solve(coeff(F,y,4)=0,a[4]);
      b4:= $\frac{1}{3}q^5 + \frac{5}{8}q^7 - \frac{5}{24}q^3 + \frac{1}{12}q$ 

  > b[5]:=solve(coeff(F,y,5)=0,a[5]);
      b5:= $-\frac{1}{20}q - \frac{13}{40}q^5 + \frac{3}{8}q^7 + \frac{1}{8}q^3 + \frac{7}{8}q^9$ 

  > b[6]:=solve(coeff(F,y,6)=0,a[6]);
      b6:= $\frac{161}{720}q^5 - \frac{389}{720}q^7 + \frac{7}{16}q^9 + \frac{21}{16}q^{11} - \frac{7}{90}q^3 + \frac{1}{30}q$ 

  > b[7]:=solve(coeff(F,y,7)=0,a[7]);
      b7:= $-\frac{1}{42}q + \frac{1}{20}q^3 - \frac{11}{80}q^5 + \frac{179}{420}q^7 - \frac{37}{40}q^9 + \frac{1}{2}q^{11} + \frac{33}{16}q^{13}$ 

  > b[8]:=solve(coeff(F,y,8)=0,a[8]);
      b8:= $-\frac{331}{10080}q^3 + \frac{73}{960}q^5 - \frac{3547}{13440}q^7 + \frac{4213}{5040}q^9 - \frac{103}{64}q^{11} + \frac{33}{64}q^{13} + \frac{1}{56}q + \frac{429}{128}q^{15}$ 

  >
  > restart:readlib(mttaylor):
      A:=0:

      b[1]:=q:b[2]:=1/2*(q^2+1)*q:b[3]:=1/6*(-
      1+2*q^2+3*q^4)*q:b[4]:=1/24*(8*q^4+15*q^6+2-5*q^2)*q:
      b[5]:=1/40*(-2+5*q^2-
      13*q^4+15*q^6+35*q^8)*q:b[6]:=1/720*(945*q^10+161*q^4+24-56*q^2-
      389*q^6+315*q^8)*q:b[7]:=1/1680*(-40+3465*q^12-
      231*q^4+716*q^6+840*q^10+84*q^2-1554*q^8)*q:b[8]:=1/40320*(-
      1324*q^2+720+135135*q^14-10641*q^6+33704*q^8+20790*q^12+3066*q^4-
      64890*q^10)*q:
      P:=normal(sum('b[k]*y^k','k'=1..8)):

```

Q:=mtaylor(P^2+y^2,y,8):

# First step: (3,4)-normalization ----- (2,3,4) jet >

```
T:=subs([x=(I*z-I*cz)/2,y=(I*z+I*cz)/(2*I)],x*P+n*Q):
S:=subs([x=(I*z-I*cz)/2,y=(I*z+I*cz)/(2*I)],P):
S1:=mtaylor(4*S/q/(q^2+1),[z,cz],7): T1:=mtaylor(101606400*T/n,
[z,cz],8):
S2:=normal(int(diff(int(diff(S1,z),z),cz),cz)):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
qq:=50803200*q^2+50803200:
T3:=collect(n*(T2-qq*S2)/6350400,[z,cz],distributed):
```

### Result check.

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=factor(normal(simplify(simplify(expand(subs([z=0,cz=0],
diff(T3,z,z,cz))))/2))));
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := I(q - 2In)(q + I)(q - I)$$

$$T_{3,0,0,0} := 0$$

**Introduce the notations.**

```

> k:=2*n*q^2+I*q+I*q^3+2*n:
ck:=2*n*q^2-I*q-I*q^3+2*n:
M:=factor(expand(k*ck));
M:=(q^2+1)^2(q^2+4n^2)

```

### Introduce new variables

```

> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):

```

### Write a mapping

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:

g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:

cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:

h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:

ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:

```

### Write the relations for the mapping.

```

> e2:=subs([z=f, cz=cf], 2*S3)+I*(g-cg):
e3:=subs([z=f, cz=cf], 2*T3)+I*(h-ch):

```

### Write the coefficients of the (3,4)-jet

```

> P[3,0,0,0]:=expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z))):
P[2,1,0,0]:=expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, cz))):
P[1,0,1,0]:=expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, z, u2))):
P[0,0,0,1]:=expand(subs([z=0, cz=0, u2=0, u3=0], diff(e2, u3))):

```

```

P1[3,0,0,0]:=subs(I=0, P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-

```

```

P1[3,0,0,0]) * I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0]) * I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0]) * I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1]) * I):

```

```

Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

```

```

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0]) * I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0]) * I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0]) * I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1]) * I):

```

**Write 17 equations.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**Solve the system.**

```

> solve(EQ)[3];
{H2I = r^2 (20 n r^2 q^11 + 5 n r^2 q^9 + 160 n^3 r^2 q^9 + 320 n^5 r^2 q^7 + 40 n^3 r^2 q^7 - 2 n r^2 q^7
+ 80 n^5 r^2 q^5 - 16 n^3 r^2 q^5 - n r^2 q^5 - 32 n^5 r^2 q^3 - 8 n^3 r^2 q^3 - 16 n^2 q^2 f2 - 16 n^5 r^2 q
- 2 q^4 f2 - 32 n^4 f2 - 32 n^4 q^6 f2 - 96 n^4 q^4 f2 - 96 n^4 q^2 f2 + 24 n^3 r^2 q^15
- 16 n^2 q^8 f2 - 48 n^2 q^4 f2 - 48 n^2 q^6 f2 + 25 n r^2 q^13 + 14 n r^2 q^15 - 6 q^8 f2 - 6 q^6 f2

```

$$\begin{aligned}
& -2q^{10}f_2 + 3nr^2q^{17} + 48n^5r^2q^{13} + 224n^5r^2q^{11} + 400n^5r^2q^9 + 200n^3r^2q^{11} \\
& + 112n^3r^2q^{13})/q, F01 = nr^2q^7 + \frac{5}{3}nr^2q^5 + \frac{1}{3}nr^2q^3 - \frac{1}{3}nr^2q - f_2, G001 = -\frac{1}{3}r( \\
& -12n^2f_2 - 4r^2n^3q - 3q^2f_2 - nr^2q^3 + 3nr^2q^{11} + 8nr^2q^9 + 12n^3r^2q^9
\end{aligned}$$

$$\begin{aligned}
& + 32n^3r^2q^7 + 6nr^2q^7 + 24n^3r^2q^5 - 12n^2q^2f_2 - 3q^4f_2)/q, g001 = 0, H4 = 0, \\
& h4 = 0, G3 = 0, g3 = 0, H101 = 0, h101 = 0, g11 = \\
& -r^3q^7 - \frac{16}{3}r^3n^2q^7 - 4r^3n^2q^5 - 2r^3q^9n^2 + \frac{2}{3}r^3n^2q - \frac{4}{3}r^3q^9 + \frac{1}{6}r^3q^3 - \frac{1}{2}r^3q^{11}, F2
\end{aligned}$$

$$= \frac{1}{12} \frac{3r^2q^9 + 12n^2r^2q^7 + 5r^2q^7 + r^2q^5 + 20n^2r^2q^5 + 4n^2r^2q^3 - r^2q^3 - 4n^2r^2q - 24}{q}$$

$$\begin{aligned}
& ,f01 = \frac{1}{12}(3r^2q^9 + 36n^2r^2q^7 + 5r^2q^7 + r^2q^5 + 60n^2r^2q^5 + 12n^2r^2q^3 - r^2q^3 \\
& - 12n^2r^2q - 24nf_2)/q, G11 = r(-8n^2f_2 - 4r^2n^3q - 2q^2f_2 - nr^2q^3 + 3nr^2q^{11}
\end{aligned}$$

$$\begin{aligned}
& + 8nr^2q^9 + 12n^3r^2q^9 + 32n^3r^2q^7 + 6nr^2q^7 + 24n^3r^2q^5 - 8n^2q^2f_2 - 2q^4f_2)/q, \\
& q = q, f_2 = f_2, n = n, r = r, H02 = \frac{8}{3}n^4r^4q + \frac{1}{6}r^4q^5 - \frac{5}{6}r^4q^9 - \frac{10}{3}r^4q^{11} - \frac{40}{3}n^4r^4q^5 \\
& - \frac{56}{3}n^2r^4q^{13} - \frac{200}{3}n^4r^4q^9 - \frac{112}{3}n^4r^4q^{11} - \frac{100}{3}n^2r^4q^{11} + \frac{8}{3}n^2r^4q^5 + \frac{4}{3}n^2r^4q^3
\end{aligned}$$

$$\begin{aligned}
& + \frac{16}{3}n^4r^4q^3 - 4n^2r^4q^{15} - 8n^4r^4q^{13} - \frac{25}{6}r^4q^{13} - \frac{7}{3}r^4q^{15} - \frac{1}{2}r^4q^{17} + \frac{1}{3}r^4q^7 \\
& - \frac{20}{3}n^2r^4q^7 - \frac{80}{3}n^2r^4q^9 - \frac{160}{3}n^4r^4q^7, h02 = \frac{2}{3}r^2(20nr^2q^{11} + 5nr^2q^9 \\
& + 160n^3r^2q^9 + 320n^5r^2q^7 + 40n^3r^2q^7 - 2nr^2q^7 + 80n^5r^2q^5 - 16n^3r^2q^5
\end{aligned}$$

$$\begin{aligned}
& - nr^2q^5 - 32n^5r^2q^3 - 8n^3r^2q^3 - 24n^2q^2f_2 - 16n^5r^2q - 3q^4f_2 - 48n^4f_2 \\
& - 48n^4q^6f_2 - 144n^4q^4f_2 - 144n^4q^2f_2 + 24n^3r^2q^{15} - 24n^2q^8f_2 - 72n^2q^4f_2 \\
& - 72n^2q^6f_2 + 25nr^2q^{13} + 14nr^2q^{15} - 9q^8f_2 - 9q^6f_2 - 3q^{10}f_2 + 3nr^2q^{17} \\
& + 48n^5r^2q^{13} + 224n^5r^2q^{11} + 400n^5r^2q^9 + 200n^3r^2q^{11} + 112n^3r^2q^{13})/q, h21 =
\end{aligned}$$

$$\begin{aligned} & \frac{8}{3}n^4 r^4 q + \frac{1}{6}r^4 q^5 - \frac{5}{6}r^4 q^9 - \frac{10}{3}r^4 q^{11} - \frac{40}{3}n^4 r^4 q^5 - \frac{56}{3}n^2 r^4 q^{13} - \frac{200}{3}n^4 r^4 q^9 \\ & - \frac{112}{3}n^4 r^4 q^{11} - \frac{100}{3}n^2 r^4 q^{11} + \frac{8}{3}n^2 r^4 q^5 + \frac{4}{3}n^2 r^4 q^3 + \frac{16}{3}n^4 r^4 q^3 - 4n^2 r^4 q^{15} \\ & - 8n^4 r^4 q^{13} - \frac{25}{6}r^4 q^{13} - \frac{7}{3}r^4 q^{15} - \frac{1}{2}r^4 q^{17} + \frac{1}{3}r^4 q^7 - \frac{20}{3}n^2 r^4 q^7 - \frac{80}{3}n^2 r^4 q^9 \\ & - \frac{160}{3}n^4 r^4 q^7 \} \end{aligned}$$

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```

>
> restart:readlib(mttaylor):
      A:=0:B:=0:
      b[1]:=q:b[2]:=1/2*(q^2+1)*q:b[3]:=1/6*(-
1+2*q^2+3*q^4)*q:b[4]:=1/24*(8*q^4+15*q^6+2-5*q^2)*q:
      b[5]:=1/40*(-2+5*q^2-
13*q^4+15*q^6+35*q^8)*q:b[6]:=1/720*(945*q^10+161*q^4+24-56*q^2-
389*q^6+315*q^8)*q:b[7]:=1/1680*(-40+3465*q^12-
231*q^4+716*q^6+840*q^10+84*q^2-1554*q^8)*q:b[8]:=1/40320*(-
1324*q^2+720+135135*q^14-10641*q^6+33704*q^8+20790*q^12+3066*q^4-
64890*q^10)*q:
      P:=sum('b[k]*y^k','k'=1..8):
      Q:=mtaylor(P^2+y^2,y,8):
      T:=subs([x=(I*z-I*cz)/2,y=(I*z+I*cz)/(2*I)],x*P+n*Q):
      S:=subs([x=(I*z-I*cz)/2,y=(I*z+I*cz)/(2*I)],P):
      S1:=mtaylor(4*S/q/(q^2+1),[z,cz],7): T1:=mtaylor(101606400*T/n,
[z,cz],8):
      S2:=normal(int(diff(int(diff(S1,z),z),cz),cz)):
      T2:=normal(int(diff(int(diff(T1,z),z),cz),cz)):
      S3:=S2:
      qq:=50803200*q^2+50803200:
      T3:=collect(n*(T2-qq*S2)/6350400,[z,cz],distributed):
      k:=2*n*q^2+I*q+I*q^3+2*n:
      ck:=2*n*q^2-I*q-I*q^3+2*n:
      M:=factor(expand(k*ck)):
      P:=qy + 1/2*(q^2 + 1)qy^2 + 1/6*(-1 + 2q^2 + 3q^4)qy^3 + 1/24*(8q^4 + 15q^6 + 2 - 5q^2)qy^4
      + 1/40*(-2 + 5q^2 - 13q^4 + 15q^6 + 35q^8)qy^5

```

$$\begin{aligned}
& + \frac{1}{720} (945 q^{10} + 161 q^4 + 24 - 56 q^2 - 389 q^6 + 315 q^8) q y^6 \\
& + \frac{1}{1680} (-40 + 3465 q^{12} - 231 q^4 + 716 q^6 + 840 q^{10} + 84 q^2 - 1554 q^8) q y^7 + \frac{1}{40320} \\
& (-1324 q^2 + 720 + 135135 q^{14} - 10641 q^6 + 33704 q^8 + 20790 q^{12} + 3066 q^4 \\
& - 64890 q^{10}) q y^8
\end{aligned}$$

$$\begin{aligned}
Q := & (q^2 + 1)y^2 + q^2 y^3 (q^2 + 1) + \left( \frac{1}{3} q^2 (-1 + 2 q^2 + 3 q^4) + \frac{1}{4} (q^2 + 1)^2 q^2 \right) y^4 \\
& + \left( \frac{1}{6} (-1 + 2 q^2 + 3 q^4) q^2 (q^2 + 1) + \frac{1}{12} q^2 (8 q^4 + 15 q^6 + 2 - 5 q^2) \right) y^5 + \left( \right. \\
& \frac{1}{20} q^2 (-2 + 5 q^2 - 13 q^4 + 15 q^6 + 35 q^8) + \frac{1}{24} (8 q^4 + 15 q^6 + 2 - 5 q^2) q^2 (q^2 + 1) \\
& \left. + \frac{1}{36} (-1 + 2 q^2 + 3 q^4)^2 q^2 \right) y^6 + \left( \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{360} (945 q^{10} + 161 q^4 + 24 - 56 q^2 - 389 q^6 + 315 q^8) q^2 \\
& + \frac{1}{40} (q^2 + 1) q^2 (-2 + 5 q^2 - 13 q^4 + 15 q^6 + 35 q^8) \\
& \left. + \frac{1}{72} (-1 + 2 q^2 + 3 q^4) q^2 (8 q^4 + 15 q^6 + 2 - 5 q^2) \right) y^7
\end{aligned}$$

$$M := (q^2 + 1)^2 (q^2 + 4 n^2)$$

```

> f2:=a: H4:= 0: g3:= 0: h4:= 0: G3:= 0: H101:= 0: g001:= 0: h101:=
0: h02:= -2/3*r^2*(-224*n^5*r^2*q^11-80*n^5*r^2*q^5-
48*n^5*r^2*q^13+8*n^3*r^2*q^3-3*n*r^2*q^17-200*n^3*r^2*q^11-
112*n^3*r^2*q^13-160*n^3*r^2*q^9-
40*n^3*r^2*q^7+32*n^5*r^2*q^3+48*n^4*f2-400*n^5*r^2*q^9-
320*n^5*r^2*q^7+3*q^4*f2+9*q^6*f2-24*n^3*r^2*q^15-20*n*r^2*q^11-
14*n*r^2*q^15+16*n^3*r^2*q^5-25*n*r^2*q^13-
5*n*r^2*q^9+16*n^5*r^2*q+9*q^8*f2+3*q^10*f2+144*n^4*q^4*f2+144*n^4*
q^2*f2+48*n^4*q^6*f2+2*n*r^2*q^7+n*r^2*q^5+24*n^2*q^8*f2+72*n^2*q^6
*f2+72*n^2*q^4*f2+24*n^2*q^2*f2)/q: h21:= -100/3*n^2*r^4*q^11-
80/3*n^2*r^4*q^9-40/3*n^4*r^4*q^5-8*n^4*r^4*q^13-4*n^2*r^4*q^15-
160/3*n^4*r^4*q^7-56/3*n^2*r^4*q^13-5/6*r^4*q^9-
20/3*n^2*r^4*q^7+8/3*n^2*r^4*q^5+4/3*n^2*r^4*q^3+8/3*n^4*r^4*q-
7/3*r^4*q^15+16/3*n^4*r^4*q^3-200/3*n^4*r^4*q^9-112/3*n^4*r^4*q^11-
1/2*r^4*q^17-10/3*r^4*q^11+1/3*r^4*q^7+1/6*r^4*q^5-25/6*r^4*q^13:
F01:= n*r^2*q^7+5/3*n*r^2*q^5+1/3*n*r^2*q^3-1/3*n*r^2*q-f2: H21:=
-r^2*(-224*n^5*r^2*q^11-80*n^5*r^2*q^5-

```



$$\begin{aligned}
& 48*n^5*r^2*q^13+8*n^3*r^2*q^3-3*n*r^2*q^17-200*n^3*r^2*q^11- \\
& \quad 112*n^3*r^2*q^13-160*n^3*r^2*q^9- \\
& \quad 40*n^3*r^2*q^7+32*n^5*r^2*q^3+32*n^4*f2-400*n^5*r^2*q^9- \\
& 320*n^5*r^2*q^7+2*q^4*f2+6*q^6*f2-24*n^3*r^2*q^15-20*n*r^2*q^11- \\
& \quad 14*n*r^2*q^15+16*n^3*r^2*q^5-25*n*r^2*q^13- \\
& 5*n*r^2*q^9+16*n^5*r^2*q+6*q^8*f2+2*q^10*f2+96*n^4*q^4*f2+96*n^4*q^ \\
& 2*f2+32*n^4*q^6*f2+2*n*r^2*q^7+n*r^2*q^5+16*n^2*q^8*f2+48*n^2*q^6*f \\
& \quad 2+48*n^2*q^4*f2+16*n^2*q^2*f2)/q: f01:= -1/12*(12*n^2*r^2*q- \\
& \quad 12*n^2*r^2*q^3-60*n^2*r^2*q^5-36*n^2*r^2*q^7+24*n*f2-r^2*q^5- \\
& \quad 5*r^2*q^7+r^2*q^3-3*r^2*q^9)/q: F2:= -1/12*(-3*r^2*q^9- \\
& \quad 12*n^2*r^2*q^7-5*r^2*q^7-20*n^2*r^2*q^5-r^2*q^5- \\
& \quad 4*n^2*r^2*q^3+r^2*q^3+4*n^2*r^2*q+24*n*f2)/q: G001:= 1/3*r*(- \\
& 12*n^3*r^2*q^9-32*n^3*r^2*q^7+3*q^4*f2-3*n*r^2*q^11-24*n^3*r^2*q^5- \\
& \quad 8*n*r^2*q^9+3*q^2*f2- \\
& 6*n*r^2*q^7+12*n^2*q^2*f2+n*r^2*q^3+12*n^2*f2+4*r^2*n^3*q)/q: H02:= \\
& \quad -100/3*n^2*r^4*q^11-80/3*n^2*r^4*q^9-40/3*n^4*r^4*q^5- \\
& 8*n^4*r^4*q^13-4*n^2*r^4*q^15-160/3*n^4*r^4*q^7-56/3*n^2*r^4*q^13- \\
& \quad 5/6*r^4*q^9- \\
& \quad 20/3*n^2*r^4*q^7+8/3*n^2*r^4*q^5+4/3*n^2*r^4*q^3+8/3*n^4*r^4*q- \\
& 7/3*r^4*q^15+16/3*n^4*r^4*q^3-200/3*n^4*r^4*q^9-112/3*n^4*r^4*q^11- \\
& \quad 1/2*r^4*q^17-10/3*r^4*q^11+1/3*r^4*q^7+1/6*r^4*q^5-25/6*r^4*q^13: \\
& g11:= -4*r^3*n^2*q^5-16/3*r^3*n^2*q^7+2/3*r^3*n^2*q-2*r^3*q^9*n^2- \\
& \quad 1/2*r^3*q^11-4/3*r^3*q^9-r^3*q^7+1/6*r^3*q^3: G11:= -r*(- \\
& 12*n^3*r^2*q^9-32*n^3*r^2*q^7+2*q^4*f2-3*n*r^2*q^11-24*n^3*r^2*q^5- \\
& \quad 8*n*r^2*q^9+2*q^2*f2- \\
& 6*n*r^2*q^7+8*n^2*q^2*f2+n*r^2*q^3+8*n^2*f2+4*r^2*n^3*q)/q:
\end{aligned}$$

### Introduce new variables

```

> w2:=u2+I*(z*c2+B*z^2*c2): cw2:=u2-I*(z*c2+B*z^2*c2):
w3:=u3+I*(z^2*c2+c2^2*z+(C1+I*C2)*z^3*c2^2+(C1-I*C2)*c2^3*z^2):
cw3:=u3-I*(z^2*c2+c2^2*z+(C1+I*C2)*z^3*c2^2+(C1-I*C2)*c2^3*z^2):

```

### Write a mapping

```

> f:=
r*c*k*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

```

```

cf:=
r*k*c2+
(F2-I*f2)*c2^2+(F01-I*f01)*cw2+
(F3-I*f3)*c2^3+(F11-I*f11)*c2*cw2+(F001-I*f001)*cw3:

```

```

g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

```

```

cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

```

```

h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

```

```

ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

**Write the relations for the mapping.**

```

> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):

```

**Write the coefficients of the (4,5)-jet.**

```

> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):

P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):

Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):

```

```

Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write 26 equations.**

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Solve the system.**

```
> sol:=solve(EQ):
```

**Here is the solution for n=0.**

```
> sol[6];
```

$$\{G101 = -\frac{1}{16}q^{14}r^4 - a^2 - \frac{29}{72}q^{10}r^4 - \frac{1}{144}q^4r^4 - \frac{13}{144}q^6r^4 - \frac{7}{24}q^8r^4 - \frac{37}{144}q^{12}r^4,$$

$$f3 = -\frac{1}{144} \frac{9q^{14}r^4 + 144a^2 + 58q^{10}r^4 + q^4r^4 + 13q^6r^4 + 42q^8r^4 + 37q^{12}r^4}{r q (q^2 + 1)},$$

$$H011 = -r^3 q^{14} a - \frac{14}{3} r^3 q^{10} a - \frac{11}{3} r^3 q^{12} a + \frac{1}{3} r^3 q^6 a - 2 r^3 q^8 a + \frac{1}{3} r^3 q^4 a,$$

$$g101 = -\frac{1}{2} r^2 q^8 a + \frac{1}{6} r^2 q^2 a - \frac{5}{6} r^2 q^6 a - \frac{1}{6} r^2 q^4 a,$$

$$h201 = -\frac{1}{2} r^3 q^{14} a - \frac{7}{3} r^3 q^{10} a - \frac{11}{6} r^3 q^{12} a + \frac{1}{6} r^3 q^6 a - r^3 q^8 a + \frac{1}{6} r^3 q^4 a,$$

$$G02 = -\frac{1}{2} r^2 q^8 a + \frac{1}{6} r^2 q^2 a - \frac{5}{6} r^2 q^6 a - \frac{1}{6} r^2 q^4 a, n = 0, q = q, B = B, H201 = -\frac{1}{144}$$

$$(q^4 + 2q^2 + 1) r q^2$$

$$(9q^{14} r^4 + 144a^2 + 58q^{10} r^4 + q^4 r^4 + 13q^6 r^4 + 42q^8 r^4 + 37q^{12} r^4), a = a,$$

$$g21 = \frac{3}{2} r^2 q^8 a - \frac{1}{2} r^2 q^2 a + \frac{5}{2} r^2 q^6 a + \frac{1}{2} r^2 q^4 a, r = r,$$

$$C1 = \frac{1}{24} \frac{9q^{14} r^4 + 144a^2 + 58q^{10} r^4 + q^4 r^4 + 13q^6 r^4 + 42q^8 r^4 + 37q^{12} r^4}{q^2 r^2 (q^2 + 1)^2}, F11 = 0,$$

$$F3 = -\frac{1}{2} r q^5 a + \frac{1}{6} r a q - \frac{1}{3} r a q^3,$$

$$F001 = -\frac{1}{288} \frac{9q^{14} r^4 + 144a^2 + 58q^{10} r^4 + q^4 r^4 + 13q^6 r^4 + 42q^8 r^4 + 37q^{12} r^4}{r q (q^2 + 1)},$$

$$f001 = \frac{1}{4} r q^5 a - \frac{1}{12} r a q + \frac{1}{6} r a q^3, g4 = 0, G4 = 0, h5 = 0, H5 = 0, h011 = 0,$$

$$f11 = -r q^5 a + \frac{1}{3} r a q - \frac{2}{3} r a q^3, g02 = \frac{1}{16} q^{16} r^4 - \frac{5}{48} q^{14} r^4 - \frac{77}{72} q^{12} r^4 - \frac{143}{72} q^{10} r^4$$

$$- \frac{73}{48} q^8 r^4 - \frac{67}{144} q^6 r^4 + r^2 B q^6 + 2 r^2 B q^4 - \frac{1}{36} q^4 r^4 + r^2 B q^2 - 4 a^2, h12 = \frac{1}{24} r q^2 ($$

$$96 q^{14} r^4 + 9 r^4 q^{20} + 48 q^4 a^2 + 48 a^2 + 96 a^2 q^2 + 100 q^{16} r^4 + 48 q^{18} r^4 - 16 q^{10} r^4$$

$$+ q^4 r^4 - 12 q^8 r^4 + 30 q^{12} r^4), H3I = -\frac{1}{36} r q^2 (96 q^{14} r^4 + 9 r^4 q^{20} - 144 q^4 a^2 - 144 a^2 - 288 a^2 q^2 + 100 q^{16} r^4 + 48 q^{18} r^4 - 16 q^{10} r^4 + q^4 r^4 - 12 q^8 r^4 + 30 q^{12} r^4)$$

$$, G2I = 2 a^2 - \frac{1}{4} q^{16} r^4 - \frac{31}{36} q^{12} r^4 - \frac{1}{36} q^4 r^4 + \frac{1}{18} q^6 r^4 + \frac{1}{4} q^8 r^4 - \frac{1}{9} q^{10} r^4 - \frac{5}{6} q^{14} r^4,$$

$$C2 = 0, h3I = 2 r^3 q^{14} a + 4 r^3 q^8 a - \frac{2}{3} r^3 q^4 a + \frac{22}{3} r^3 q^{12} a + \frac{28}{3} r^3 q^{10} a - \frac{2}{3} r^3 q^6 a,$$

$$H12 = r^3 q^{14} a + 2 r^3 q^8 a - \frac{1}{3} r^3 q^4 a + \frac{11}{3} r^3 q^{12} a + \frac{14}{3} r^3 q^{10} a - \frac{1}{3} r^3 q^6 a \}$$

> CC1 :=

factor(1/24\*(9\*q^14\*r^4+144\*a^2+58\*q^10\*r^4+q^4\*r^4+13\*q^6\*r^4+42\*q^8\*r^4+37\*q^12\*r^4)/(q^2\*r^2\*(q^2+1)^2));

$$CCI := \frac{1}{24} \frac{9 q^{14} r^4 + 144 a^2 + 58 q^{10} r^4 + q^4 r^4 + 13 q^6 r^4 + 42 q^8 r^4 + 37 q^{12} r^4}{q^2 r^2 (q^2 + 1)^2}$$

**Here is the solution for general n:**

> sol[7];

$$\{H01I = \frac{1}{18} (816 n^5 r^2 q^{11} - 1488 n^5 r^2 q^5 + 2320 n^5 r^2 q^{13} + 8 n^3 r^2 q^3 + 145 n r^2 q^{17} - 1080 n^3 r^2 q^{11} + 408 n^3 r^2 q^{13} - 1496 n^3 r^2 q^9 - 744 n^3 r^2 q^7 - 240 n^5 r^2 q^3 - 2160 n^5 r^2 q^9 - 2992 n^5 r^2 q^7 + 1160 n^3 r^2 q^{15} - 576 n^4 q^4 a + 96 n^4 q^2 a$$

$$- 1344 n^4 q^6 a - 672 n^2 q^8 a - 288 n^2 q^6 a + 48 n^2 q^4 a + 48 n^2 q^2 a - 187 n r^2 q^{11} + 51 n r^2 q^{15} - 120 n^3 r^2 q^5 - 135 n r^2 q^{13} - 84 q^{10} a - 36 q^8 a + 6 q^6 a + 6 q^4 a + 96 n^4 a - 93 n r^2 q^9 + 16 n^5 r^2 q - 15 n r^2 q^7 + n r^2 q^5 - 144 n^2 q^{12} a - 1056 n^4 q^8 a - 528 n^2 q^{10} a - 288 n^4 q^{10} a - 66 q^{12} a + 18 q^{21} n r^2 - 18 q^{14} a + 288 n^5 q^{17} r^2$$

$$+ 1392 n^5 q^{15} r^2 + 696 n^3 q^{17} r^2 + 87 n q^{19} r^2 + 144 n^3 q^{19} r^2) r^3, H20I = -\frac{1}{144} ((144 n r^2 q^5 a + 141 q^{18} r^4 - 6912 q^7 n^3 r^2 a - 864 n r^2 q^9 a + 3636 q^{16} n^2 r^4 + 22320 q^{14} n^4 r^4 + 1152 n^2 q^6 a^2 - 432 n r^2 q^{15} a - 6912 n^5 r^2 q^{11} a$$

$$\begin{aligned}
& - 3456 n^3 r^2 q^{13} a + 4608 n^4 q^2 a^2 + 1152 n^2 a^2 q^2 - 1584 n r^2 q^{13} a - 2016 n r^2 q^{11} a \\
& - 16128 n^3 r^2 q^9 a - 12672 n^3 r^2 q^{11} a - 32256 n^5 r^2 q^7 a + 2304 n^5 r^2 q a \\
& + 2304 n^5 r^2 q^3 a - 25344 n^5 r^2 q^9 a - 13824 n^5 r^2 q^5 a + 4608 n^6 r^4 q^{18} + 144 q^8 a^2 \\
& + 2304 n^4 r^4 q^{20} + 288 n^2 r^4 q^{22} + 2304 n^4 q^4 a^2 + 144 q^4 a^2 + 2304 n^4 a^2
\end{aligned}$$

$$\begin{aligned}
& + 448 n^6 r^4 q^2 - 18112 n^6 r^4 q^8 + 2880 n^6 r^4 q^{10} + 24720 n^4 r^4 q^{16} + 24000 n^6 r^4 q^{16} \\
& + 40128 n^6 r^4 q^{12} + 47680 n^6 r^4 q^{14} - 10560 n^6 r^4 q^6 + 1572 n^2 r^4 q^{20} + 3420 n^2 r^4 q^{18} \\
& + 12144 n^4 r^4 q^{18} + 288 q^6 a^2 + 2304 n^2 q^4 a^2 + 55 r^4 q^{20} + 9 r^4 q^{22} - 960 n^6 q^4 r^4 \\
& + 60 n^2 r^4 q^8 + 36 n^2 r^4 q^6 - 240 n^4 r^4 q^6 + 240 n^4 r^4 q^4 - 108 q^{10} n^2 r^4 + 108 q^{12} n^2 r^4
\end{aligned}$$

$$\begin{aligned}
& - 4176 q^8 n^4 r^4 - 6576 q^{10} n^4 r^4 + 155 q^{14} r^4 + q^8 r^4 + 15 q^{10} r^4 + 144 n r^2 q^7 a \\
& + 69 q^{12} r^4 + 1152 q^5 n^3 r^2 a + 1152 q^3 n^3 r^2 a + 4560 q^{12} n^4 r^4 + 195 q^{16} r^4 \\
& + 1740 q^{14} n^2 r^4) r) / q^2, GI01 = - \frac{1}{144} (- 144 n r^2 q^5 a + 144 a^2 q^2 - 1728 q^7 n^3 r^2 a
\end{aligned}$$

$$\begin{aligned}
& - 432 n r^2 q^9 a + 576 n^2 a^2 + 288 q^{16} n^2 r^4 + 1152 q^{14} n^4 r^4 - 288 n^2 r^4 q^8 \\
& - 64 n^2 r^4 q^6 - 1824 n^4 r^4 q^6 + 32 n^2 r^4 q^4 - 464 n^4 r^4 q^4 + 112 n^4 r^4 q^2 + 128 q^{10} n^2 r^4 \\
& + 992 q^{12} n^2 r^4 - 416 q^8 n^4 r^4 + 3376 q^{10} n^4 r^4 + 37 q^{14} r^4 + q^6 r^4 + 13 q^8 r^4 + 42 q^{10} r^4 \\
& + 144 n r^2 q^3 a - 720 n r^2 q^7 a + 58 q^{12} r^4 - 2880 q^5 n^3 r^2 a - 576 q^3 n^3 r^2 a
\end{aligned}$$

$$\begin{aligned}
& + 576 q n^3 r^2 a + 3696 q^{12} n^4 r^4 + 9 q^{16} r^4 + 960 q^{14} n^2 r^4) / q^2, h31 = 4 n r^5 q^9 \\
& + 32 n^3 r^5 q^7 + \frac{128}{3} n^3 r^5 q^9 - 16 n r^5 q^{19} - \frac{8}{3} n^3 r^5 q^3 - 128 n^3 r^5 q^{17} - 32 n r^5 q^{15} \\
& - \frac{100}{3} n r^5 q^{17} - 10 n r^5 q^{13} - 24 n^3 r^5 q^{19} + 64 n^4 r^3 q^4 a + 2 r^3 q^{14} a + 4 r^3 q^8 a
\end{aligned}$$

$$\begin{aligned}
& - \frac{2}{3} r^3 q^4 a + \frac{22}{3} r^3 q^{12} a + \frac{28}{3} r^3 q^{10} a + \frac{256}{3} n^5 r^5 q^7 - 256 n^3 r^5 q^{13} - \frac{16}{3} n^5 r^5 q \\
& - 160 n^5 r^5 q^9 - 512 n^5 r^5 q^{11} - \frac{32}{3} n^4 r^3 q^2 a - \frac{16}{3} n^2 r^3 q^4 a + \frac{224}{3} n^2 r^3 q^8 a
\end{aligned}$$

$$\begin{aligned}
& + \frac{176}{3} n^2 r^3 q^{10} a + \frac{352}{3} n^4 r^3 q^8 a + 32 n^4 r^3 q^{10} a + 16 n^2 r^3 q^{12} a + \frac{448}{3} n^4 r^3 q^6 a \\
& + 32 n^2 r^3 q^6 a - \frac{32}{3} n^4 r^3 a - 3 n r^5 q^{21} - 48 n^5 r^5 q^{17} - 256 n^5 r^5 q^{15} + \frac{16}{3} n r^5 q^{11} \\
& - 80 n^3 r^5 q^{11} + 64 n^5 r^5 q^5 - \frac{1600}{3} n^5 r^5 q^{13} - \frac{800}{3} n^3 r^5 q^{15} - \frac{2}{3} r^3 q^6 a - \frac{16}{3} n^2 r^3 a q^2 \\
& - \frac{1}{3} n r^5 q^5, G21 = \frac{1}{36} (-84 n r^2 q^5 a + 72 a^2 q^2 - 9 q^{18} r^4 - 1008 q^7 n^3 r^2 a \\
& - 252 n r^2 q^9 a + 288 n^2 a^2 + 180 q^{16} n^2 r^4 + 864 q^{14} n^4 r^4 - 180 n^2 r^4 q^8 - 40 n^2 r^4 q^6 \\
& - 864 n^4 r^4 q^6 + 20 n^2 r^4 q^4 - 192 n^4 r^4 q^4 + 96 n^4 r^4 q^2 + 80 q^{10} n^2 r^4 + 620 q^{12} n^2 r^4 \\
& + 384 q^8 n^4 r^4 + 2976 q^{10} n^4 r^4 - 31 q^{14} r^4 - q^6 r^4 + 2 q^8 r^4 + 9 q^{10} r^4 + 84 n r^2 q^3 a \\
& - 420 n r^2 q^7 a - 4 q^{12} r^4 - 1680 q^5 n^3 r^2 a - 336 q^3 n^3 r^2 a + 336 q n^3 r^2 a \\
& + 2880 q^{12} n^4 r^4 - 30 q^{16} r^4 + 600 q^{14} n^2 r^4) / q^2, f3 = \frac{1}{144} (144 q^{14} n^2 r^4 \\
& + 360 n^2 r^4 q^6 + 124 n^2 r^4 q^4 + 588 q^{12} n^2 r^4 + 28 n^2 r^4 q^2 + 760 n^2 r^4 q^8 \\
& + 940 q^{10} n^2 r^4 - 37 q^{12} r^4 - q^4 r^4 - 13 q^6 r^4 - 144 a^2 - 42 q^8 r^4 - 58 q^{10} r^4 - 9 q^{14} r^4) \\
& / (r q (q^2 + 1)), g21 = - \frac{10}{9} r^4 q^9 n + 10 r^4 n^3 q^5 + \frac{3}{2} r^2 q^8 a - 2 r^2 n^2 a - \frac{1}{2} r^2 q^2 a \\
& - \frac{5}{2} r^4 q^{15} n - \frac{5}{18} r^4 q^3 n + \frac{5}{2} r^2 q^6 a + \frac{1}{2} r^2 q^4 a - \frac{310}{9} r^4 n^3 q^9 + \frac{20}{9} r^4 n^3 q^3 - \frac{10}{9} r^4 n^3 q \\
& - \frac{100}{3} r^4 q^{11} n^3 - \frac{25}{3} r^4 q^{13} n + \frac{5}{9} r^4 q^5 n + 2 r^2 n^2 q^2 a + 10 r^2 n^2 q^4 a + 6 n^2 r^2 q^6 a \\
& + \frac{5}{2} r^4 q^7 n - \frac{40}{9} r^4 n^3 q^7 - 10 n^3 r^4 q^{13} - \frac{155}{18} r^4 q^{11} n, g101 = \frac{1}{36} (18 n r^2 q^{15} \\
& + 72 n^3 r^2 q^{13} + 51 n r^2 q^{13} + 204 n^3 r^2 q^{11} + 25 n r^2 q^{11} + 100 n^3 r^2 q^9 - 50 n r^2 q^9 \\
& - 18 q^8 a - 200 n^3 r^2 q^7 - 60 n r^2 q^7 - 72 n^2 q^6 a - 30 q^6 a - 17 n r^2 q^5 - 240 n^3 r^2 q^5 \\
& - 120 n^2 q^4 a - 6 q^4 a + n r^2 q^3 - 68 n^3 r^2 q^3 - 24 n^2 q^2 a + 6 q^2 a + 4 r^2 n^3 q \\
& + 24 n^2 a) r^2, H31 = \frac{1}{36} r (144 n r^2 q^5 a - 96 q^{18} r^4 - 6912 q^7 n^3 r^2 a - 864 n r^2 q^9 a
\end{aligned}$$

$$\begin{aligned}
& + 2688 q^{16} n^2 r^4 + 26112 q^{14} n^4 r^4 - 9 r^4 q^{24} + 1152 n^2 q^6 a^2 - 432 n r^2 q^{15} a \\
& - 6912 n^5 r^2 q^{11} a - 3456 n^3 r^2 q^{13} a + 4608 n^4 q^2 a^2 + 1152 n^2 a^2 q^2 \\
& - 1584 n r^2 q^{13} a - 2016 n r^2 q^{11} a - 16128 n^3 r^2 q^9 a - 12672 n^3 r^2 q^{11} a \\
& - 32256 n^5 r^2 q^7 a + 2304 n^5 r^2 q a + 2304 n^5 r^2 q^3 a - 25344 n^5 r^2 q^9 a \\
\\
& - 13824 n^5 r^2 q^5 a + 5184 n^6 r^4 q^{18} + 144 q^8 a^2 + 2448 n^4 r^4 q^{20} + 252 n^2 r^4 q^{22} \\
& + 2304 n^4 q^4 a^2 + 144 q^4 a^2 + 2304 n^4 a^2 + 576 n^6 r^4 q^2 - 9216 n^6 r^4 q^8 \\
& + 17280 n^6 r^4 q^{10} + 27200 n^4 r^4 q^{16} + 27648 n^6 r^4 q^{16} + 55296 n^6 r^4 q^{12} \\
& + 57600 n^6 r^4 q^{14} - 6912 n^6 r^4 q^6 + 1344 n^2 r^4 q^{20} + 2800 n^2 r^4 q^{18} + 13056 n^4 r^4 q^{18} \\
\\
& + 288 q^6 a^2 + 2304 n^2 q^4 a^2 - 100 r^4 q^{20} - 48 r^4 q^{22} + 28 n^2 r^4 q^6 + 272 n^4 r^4 q^4 \\
& - 336 q^{10} n^2 r^4 - 448 q^{12} n^2 r^4 - 3264 q^8 n^4 r^4 - 4352 q^{10} n^4 r^4 + 16 q^{14} r^4 - q^8 r^4 \\
& + 144 n r^2 q^7 a + 12 q^{12} r^4 + 1152 q^5 n^3 r^2 a + 1152 q^3 n^3 r^2 a + 8160 q^{12} n^4 r^4 \\
\\
& - 30 q^{16} r^4 + 840 q^{14} n^2 r^4) / q^2, H12 = 2 n r^5 q^9 + 16 n^3 r^5 q^7 + \frac{64}{3} n^3 r^5 q^9 \\
& - 8 n r^5 q^{19} - \frac{4}{3} n^3 r^5 q^3 - 64 n^3 r^5 q^{17} - 16 n r^5 q^{15} - \frac{50}{3} n r^5 q^{17} - 5 n r^5 q^{13} \\
\\
& - 12 n^3 r^5 q^{19} + 32 n^4 r^3 q^4 a + r^3 q^{14} a + 2 r^3 q^8 a - \frac{1}{3} r^3 q^4 a + \frac{11}{3} r^3 q^{12} a \\
& + \frac{14}{3} r^3 q^{10} a + \frac{128}{3} n^5 r^5 q^7 - 128 n^3 r^5 q^{13} - \frac{8}{3} n^5 r^5 q - 80 n^5 r^5 q^9 - 256 n^5 r^5 q^{11} \\
\\
& - \frac{16}{3} n^4 r^3 q^2 a - \frac{8}{3} n^2 r^3 q^4 a + \frac{112}{3} n^2 r^3 q^8 a + \frac{88}{3} n^2 r^3 q^{10} a + \frac{176}{3} n^4 r^3 q^8 a \\
& + 16 n^4 r^3 q^{10} a + 8 n^2 r^3 q^{12} a + \frac{224}{3} n^4 r^3 q^6 a + 16 n^2 r^3 q^6 a - \frac{16}{3} n^4 r^3 a - \frac{3}{2} n r^5 q^{21} \\
\\
& - 24 n^5 r^5 q^{17} - 128 n^5 r^5 q^{15} + \frac{8}{3} n r^5 q^{11} - 40 n^3 r^5 q^{11} + 32 n^5 r^5 q^5 - \frac{800}{3} n^5 r^5 q^{13} \\
& - \frac{400}{3} n^3 r^5 q^{15} - \frac{1}{3} r^3 q^6 a - \frac{8}{3} n^2 r^3 a q^2 - \frac{1}{6} n r^5 q^5, q = q, n = n, B = B, G02 =
\end{aligned}$$



$$\begin{aligned}
& -\frac{23}{27}r^4q^9n - \frac{46}{9}r^4n^3q^5 - \frac{1}{2}r^2q^8a + \frac{2}{3}r^2n^2a + \frac{1}{6}r^2q^2a + \frac{1}{2}r^4q^{15}n + \frac{1}{27}r^4q^3n \\
& - \frac{5}{6}r^2q^6a - \frac{1}{6}r^2q^4a + \frac{112}{27}r^4n^3q^9 - \frac{38}{27}r^4n^3q^3 + \frac{4}{27}r^4n^3q + 6r^4q^{11}n^3 \\
& + \frac{3}{2}r^4q^{13}n - \frac{19}{54}r^4q^5n - \frac{2}{3}r^2n^2q^2a - \frac{10}{3}r^2n^2q^4a - 2n^2r^2q^6a - \frac{23}{18}r^4q^7n \\
& - \frac{92}{27}r^4n^3q^7 + 2n^3r^4q^{13} + \frac{28}{27}r^4q^{11}n,
\end{aligned}$$

$$\begin{aligned}
C2 = & -\frac{1}{6}nr^2q^7 - \frac{19}{54}nr^2q^5 - \frac{11}{54}nr^2q^3 - \frac{1}{54}nr^2q, h201 = \frac{1}{36}(816n^5r^2q^{11} \\
& - 1488n^5r^2q^5 + 2320n^5r^2q^{13} + 8n^3r^2q^3 + 145nr^2q^{17} - 1080n^3r^2q^{11} \\
& + 408n^3r^2q^{13} - 1496n^3r^2q^9 - 744n^3r^2q^7 - 240n^5r^2q^3 - 2160n^5r^2q^9
\end{aligned}$$

$$\begin{aligned}
& - 2992n^5r^2q^7 + 1160n^3r^2q^{15} - 576n^4q^4a + 96n^4q^2a - 1344n^4q^6a \\
& - 672n^2q^8a - 288n^2q^6a + 48n^2q^4a + 48n^2q^2a - 187nr^2q^{11} + 51nr^2q^{15} \\
& - 120n^3r^2q^5 - 135nr^2q^{13} - 84q^{10}a - 36q^8a + 6q^6a + 6q^4a + 96n^4a \\
& - 93nr^2q^9 + 16n^5r^2q - 15nr^2q^7 + nr^2q^5 - 144n^2q^{12}a - 1056n^4q^8a
\end{aligned}$$

$$\begin{aligned}
& - 528n^2q^{10}a - 288n^4q^{10}a - 66q^{12}a + 18q^{21}nr^2 - 18q^{14}a + 288n^5q^{17}r^2 \\
& + 1392n^5q^{15}r^2 + 696n^3q^{17}r^2 + 87nr^{19}r^2 + 144n^3q^{19}r^2)r^3, a = a, h12 = \frac{1}{24}r( \\
& 48nr^2q^5a + 96q^{18}r^4 - 2304q^7n^3r^2a - 288nr^2q^9a + 1920q^{16}n^2r^4
\end{aligned}$$

$$\begin{aligned}
& + 10752q^{14}n^4r^4 + 9r^4q^{24} + 384n^2q^6a^2 - 144nr^2q^{15}a - 2304n^5r^2q^{11}a \\
& - 1152n^3r^2q^{13}a + 1536n^4q^2a^2 + 384n^2a^2q^2 - 528nr^2q^{13}a - 672nr^2q^{11}a \\
& - 5376n^3r^2q^9a - 4224n^3r^2q^{11}a - 10752n^5r^2q^7a + 768n^5r^2qa \\
& + 768n^5r^2q^3a - 8448n^5r^2q^9a - 4608n^5r^2q^5a + 1728n^6r^4q^{18} + 48q^8a^2
\end{aligned}$$

$$\begin{aligned}
& + 1008n^4r^4q^{20} + 180n^2r^4q^{22} + 768n^4q^4a^2 + 48q^4a^2 + 768n^4a^2 + 192n^6r^4q^2 \\
& - 3072n^6r^4q^8 + 5760n^6r^4q^{10} + 11200n^4r^4q^{16} + 9216n^6r^4q^{16} + 18432n^6r^4q^{12} \\
& + 19200n^6r^4q^{14} - 2304n^6r^4q^6 + 960n^2r^4q^{20} + 2000n^2r^4q^{18} + 5376n^4r^4q^{18} \\
& + 96q^6a^2 + 768n^2q^4a^2 + 100r^4q^{20} + 48r^4q^{22} + 20n^2r^4q^6 + 112n^4r^4q^4
\end{aligned}$$

$$\begin{aligned}
& - 240 q^{10} n^2 r^4 - 320 q^{12} n^2 r^4 - 1344 q^8 n^4 r^4 - 1792 q^{10} n^4 r^4 - 16 q^{14} r^4 + q^8 r^4 \\
& + 48 n r^2 q^7 a - 12 q^{12} r^4 + 384 q^5 n^3 r^2 a + 384 q^3 n^3 r^2 a + 3360 q^{12} n^4 r^4 + 30 q^{16} r^4 \\
& + 600 q^{14} n^2 r^4) / q^2, r = r, CI = \frac{1}{72} (52 n^2 r^4 q^2 + 1636 q^{10} n^2 r^4 + 1476 q^{12} n^2 r^4
\end{aligned}$$

$$\begin{aligned}
& + 424 n^2 r^4 q^8 - 264 n^2 r^4 q^6 - 44 n^2 r^4 q^4 + 432 q^{14} n^2 r^4 - 288 n r^2 q^3 a \\
& - 1440 n r^2 q^5 a + 288 n r^2 q a - 864 n r^2 q^7 a + 27 q^{14} r^4 + 432 a^2 + 126 q^8 r^4 \\
& + 39 q^6 r^4 + 174 q^{10} r^4 + 111 q^{12} r^4 + 3 q^4 r^4) / (r^2 q^2 (q^2 + 1)^2), g4 = 0, G4 = 0,
\end{aligned}$$

$$\begin{aligned}
h5 = 0, H5 = 0, h011 = 0, F001 = - \frac{1}{288} (- 480 n r^2 q^5 a + 296 n^2 r^4 q^8 + 20 n^2 r^4 q^2 \\
+ 24 n^2 r^4 q^6 + 20 n^2 r^4 q^4 + 644 q^{10} n^2 r^4 + 516 q^{12} n^2 r^4 + 9 q^{14} r^4 + 13 q^6 r^4 \\
+ 42 q^8 r^4 + 58 q^{10} r^4 - 96 n r^2 q^3 a - 288 n r^2 q^7 a + 37 q^{12} r^4 + 96 n r^2 q a + q^4 r^4
\end{aligned}$$

$$\begin{aligned}
& + 144 q^{14} n^2 r^4 + 144 a^2) / (q r (q^2 + 1)), f11 = - \frac{1}{9} r^3 q^6 n + \frac{7}{54} r^3 n q^2 + \frac{5}{2} r^3 q^{10} n \\
& + r^3 q^{12} n - \frac{1}{9} n r^3 q^4 + \frac{44}{27} r^3 q^8 n - r q^5 a - \frac{2}{3} r a q^3 + \frac{1}{3} r a q, f001 = - \frac{1}{144} ( \\
36 n r^4 q^{16} + 288 n^3 r^4 q^{14} + 111 n r^4 q^{14} + 87 n r^4 q^{12} + 924 n^3 r^4 q^{12} + 844 n^3 r^4 q^{10}
\end{aligned}$$

$$\begin{aligned}
& - 42 n r^4 q^{10} - 36 r^2 q^9 a - 78 n r^4 q^8 - 104 n^3 r^4 q^8 - 432 q^7 n^2 r^2 a - 60 r^2 q^7 a \\
& - 456 n^3 r^4 q^6 - 21 n r^4 q^6 - 12 r^2 q^5 a - 720 q^5 n^2 r^2 a + 3 n r^4 q^4 - 116 n^3 r^4 q^4 \\
& - 144 q^3 r^2 n^2 a + 12 r^2 q^3 a + 28 n^3 r^4 q^2 + 144 q n^2 r^2 a + 144 n a^2) / (q^2 r (q^2 + 1)
\end{aligned}$$

$$), F11 = - \frac{1}{27} r (- 6 n r^2 q^5 + 7 n r^2 q + 135 n r^2 q^9 + 54 n r^2 q^{11} - 6 n r^2 q^3 + 88 n r^2 q^7$$

$$\begin{aligned}
& - 54 q^4 a - 36 q^2 a + 18 a) n, g02 = \frac{1}{144} (336 n r^2 q^5 a - 576 a^2 q^2 + 576 r^2 B n^2 q^2 \\
& + 9 q^{18} r^4 + 4032 q^7 n^3 r^2 a + 1008 n r^2 q^9 a + 144 r^2 B q^8 + 576 r^2 B n^2 q^6
\end{aligned}$$

$$\begin{aligned}
& - 2304 n^2 a^2 - 360 q^{16} n^2 r^4 - 1584 q^{14} n^4 r^4 - 648 n^2 r^4 q^8 - 232 n^2 r^4 q^6 \\
& + 912 n^4 r^4 q^6 - 64 n^2 r^4 q^4 + 144 n^4 r^4 q^4 - 192 n^4 r^4 q^2 - 1552 q^{10} n^2 r^4 \\
& - 2128 q^{12} n^2 r^4 - 1632 q^8 n^4 r^4 - 6048 q^{10} n^4 r^4 - 154 q^{14} r^4 - 4 q^6 r^4 - 67 q^8 r^4 \\
& - 219 q^{10} r^4 - 336 n r^2 q^3 a + 1680 n r^2 q^7 a - 286 q^{12} r^4 + 6720 q^5 n^3 r^2 a
\end{aligned}$$

$$\begin{aligned}
& + 1344 q^3 n^3 r^2 a - 1344 q n^3 r^2 a - 5424 q^{12} n^4 r^4 - 15 q^{16} r^4 - 1416 q^{14} n^2 r^4 \\
& + 1152 r^2 B n^2 q^4 + 144 r^2 B q^4 + 288 r^2 B q^6) / q^2, F3 = - \frac{1}{72} (60 r^2 q^7 a - 7 n r^4 q^4 \\
& - 31 n r^4 q^6 - 90 n r^4 q^8 - 190 n r^4 q^{10} - 235 n r^4 q^{12} - 147 n r^4 q^{14} + 52 n^3 r^4 q^4 \\
& + 4 n^3 r^4 q^2 - 36 n r^4 q^{16} + 148 n^3 r^4 q^{10} + 36 n^3 r^4 q^{12} + 232 n^3 r^4 q^8 + 168 n^3 r^4 q^6 \\
& + 12 r^2 q^5 a - 12 r^2 q^3 a - 48 q n^2 r^2 a + 48 q^3 r^2 n^2 a + 36 r^2 q^9 a + 240 q^5 n^2 r^2 a \\
& + 144 q^7 n^2 r^2 a - 144 n a^2) / (q^2 r (q^2 + 1)) \}
\end{aligned}$$

```
> CC2:=factor( -1/6*n*r^2*q^7-19/54*n*r^2*q^5-11/54*n*r^2*q^3-
1/54*n*r^2*q);
```

```

CC1:=
1/72*(52*n^2*r^4*q^2+1636*q^10*n^2*r^4+1476*q^12*n^2*r^4+424*n^2*r^
4*q^8-264*n^2*r^4*q^6-44*n^2*r^4*q^4+432*q^14*n^2*r^4-
288*n*r^2*q^3*a-1440*n*r^2*q^5*a+288*n*r^2*q*a-
864*n*r^2*q^7*a+27*q^14*r^4+432*a^2+126*q^8*r^4+39*q^6*r^4+174*q^10
*r^4+111*q^12*r^4+3*q^4*r^4)/(r^2*q^2*(q^2+1)^2):
CC2 := - 1/54 n r^2 q (9 q^2 + 1) (q^2 + 1)^2

```

Then we see that if  $\gamma$  is non-zero, then it is not the cubic.

```
> cc1:=subs(n=0,CC1);
cc1 := 1/72 (27 q^14 r^4 + 432 a^2 + 126 q^8 r^4 + 39 q^6 r^4 + 174 q^10 r^4 + 111 q^12 r^4 + 3 q^4 r^4) / (r^2 q^2 (q^2 + 1)^2)
```

Then we see that if  $n=0$ , then it is not the cubic as well.

>  
>

## Type IV for non-zero delta.

First step: (3,4)-normalization -----

# (2,3,4) jet

```

> restart:readlib(mttaylor):
A:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(y):
T:=exp(x+n*y):
S1:=mtaylor(4*S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=(4*T2-(1+n^2)*S2)*4:

```

## Result check.

```

> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));

```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 1 - In + n^2 - In^3 + I + In^2$$

$$T_{3,0,0,0} := 0$$

## Introduce the notations.

```

> k:=1-I*n+n^2-I*n^3+I+I*n^2:
ck:=1+I*n+n^2+I*n^3-I-I*n^2:
m:=expand(k*ck):

```

## Introduce new variables

```

> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):

```

## Write a mapping

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:

cf:=
r*k*cw2+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:

cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:

h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:

ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:

```

**Write the relations for the mapping.**

```

> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):

```

**Write the coefficients of the (3,4)-jet.**

```

> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):

P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):

Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):

```

```

Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):

Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write 17 equations.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**Solve the system.**

```

> sol:=solve(EQ):
> sol[3];

```

$$\begin{aligned}
\{H4 = 0, h4 = 0, g3 = 0, H101 = 0, h101 = 0, g001 = 0, G3 = 0, H02 = -\frac{2}{3}r^4 - \frac{14}{3}r^4 n^2 \\
+ \frac{4}{3}r^4 n - \frac{27}{2}r^4 n^4 - \frac{125}{6}r^4 n^6 + \frac{50}{3}r^4 n^5 - \frac{1}{6}r^4 n^{14} + \frac{2}{3}r^4 n^{13} - 9r^4 n^{10} + \frac{40}{3}r^4 n^9 \\
- \frac{13}{6}r^4 n^{12} + \frac{14}{3}r^4 n^{11} - \frac{55}{3}r^4 n^8 + 20r^4 n^7 + \frac{22}{3}r^4 n^3, h02 = -8r^2 f01 n^9 \\
+ 2r^2 n^{10} f01 - \frac{2}{3}r^4 + 8r^2 f01 - \frac{19}{3}r^4 n^2 + 2r^4 n - \frac{139}{6}r^4 n^4 - \frac{177}{4}r^4 n^6 + \frac{203}{6}r^4 n^5
\}
\end{aligned}$$

$$\begin{aligned}
& -\frac{23}{12}r^4n^{14} + \frac{31}{6}r^4n^{13} - \frac{1}{12}r^4n^{16} + \frac{1}{2}r^4n^{15} - \frac{63}{2}r^4n^{10} + \frac{125}{3}r^4n^9 - \frac{34}{3}r^4n^{12} \\
& + \frac{61}{3}r^4n^{11} - \frac{195}{4}r^4n^8 + \frac{295}{6}r^4n^7 + \frac{38}{3}r^4n^3 + 22r^2f01n^8 - 16r^2f01n \\
& - 56r^2f01n^3 + 40r^2f01n^2 + 74r^2f01n^4 - 72r^2f01n^5 + 62n^6r^2f01 - 40n^7r^2f01,
\end{aligned}$$

$$\begin{aligned}
g11 = & -\frac{5}{6}r^3n^6 + r^3n^5 - \frac{3}{2}r^3n^4 + r^3n^3 - \frac{7}{6}r^3n^2 + \frac{1}{3}r^3n - \frac{1}{3}r^3 - \frac{1}{6}r^3n^8 + \frac{1}{3}r^3n^7, G11 \\
= & -4rf01n - 4rf01n^3 + 6rf01n^2 + \frac{1}{6}r^3n^9 - \frac{1}{2}r^3n^8 - \frac{1}{3}r^3 + 4rf01 + \frac{2}{3}r^3n
\end{aligned}$$

$$\begin{aligned}
& + \frac{13}{6}r^3n^3 - \frac{3}{2}r^3n^2 + \frac{5}{2}r^3n^5 - \frac{5}{2}r^3n^4 + \frac{7}{6}r^3n^7 - \frac{11}{6}r^3n^6 + 2rn^4f01, f2 = \\
& \frac{1}{6}r^2n^6 - \frac{1}{3}r^2n^5 + \frac{1}{2}r^2n^4 - \frac{5}{12}r^2n^3 + \frac{1}{2}r^2n^2 - \frac{1}{6}r^2n - \frac{1}{12}n^7r^2 + f01n + \frac{1}{6}r^2 - f01,
\end{aligned}$$

$$F01 = -\frac{1}{12}r^2n^6 + \frac{1}{6}r^2n^5 - \frac{1}{3}r^2n^4 + \frac{1}{3}r^2n^3 - \frac{5}{12}r^2n^2 + \frac{1}{6}r^2n - f01n + f01 - \frac{1}{6}r^2,$$

$$h21 = -\frac{2}{3}r^4 - \frac{14}{3}r^4n^2 + \frac{4}{3}r^4n - \frac{27}{2}r^4n^4 - \frac{125}{6}r^4n^6 + \frac{50}{3}r^4n^5 - \frac{1}{6}r^4n^{14} + \frac{2}{3}r^4n^{13}$$

$$-9r^4n^{10} + \frac{40}{3}r^4n^9 - \frac{13}{6}r^4n^{12} + \frac{14}{3}r^4n^{11} - \frac{55}{3}r^4n^8 + 20r^4n^7 + \frac{22}{3}r^4n^3,$$

$$F2 = -\frac{1}{12}r^2n^6 - \frac{1}{6}r^2n^4 - \frac{1}{12}r^2n^2 + f01, r = r, f01 = f01, n = n, H21 = -8r^2f01n^9$$

$$\begin{aligned}
& + 2r^2n^{10}f01 - \frac{2}{3}r^4 + 8r^2f01 - 6r^4n^2 + 2r^4n - \frac{125}{6}r^4n^4 - \frac{75}{2}r^4n^6 + \frac{181}{6}r^4n^5 \\
& - \frac{5}{6}r^4n^{14} + \frac{17}{6}r^4n^{13} + \frac{1}{6}r^4n^{15} - \frac{67}{3}r^4n^{10} + \frac{95}{3}r^4n^9 - \frac{41}{6}r^4n^{12} + \frac{41}{3}r^4n^{11}
\end{aligned}$$

$$-\frac{115}{3}r^4n^8 + \frac{245}{6}r^4n^7 + 12r^4n^3 + 22r^2f01n^8 - 16r^2f01n - 56r^2f01n^3$$

$$+ 40r^2f01n^2 + 74r^2f01n^4 - 72r^2f01n^5 + 62n^6r^2f01 - 40n^7r^2f01, G001 =$$

$$2rf01n + 2rf01n^3 - 3rf01n^2 + \frac{1}{3}r^3n^9 - \frac{11}{12}r^3n^8 - \frac{1}{3}r^3 - 2rf01 + \frac{2}{3}r^3n + \frac{7}{3}r^3n^3$$

$$-\frac{5}{3}r^3 n^2 + 3r^3 n^5 - \frac{37}{12}r^3 n^4 + \frac{5}{3}r^3 n^7 - \frac{31}{12}r^3 n^6 - r n^4 f01 - \frac{1}{12}r^3 n^{10}\}$$

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```

> restart:readlib(mttaylor):
A:=0:B:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(y):
T:=exp(x+n*y):
S1:=mtaylor(4*S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=(4*T2-(1+n^2)*S2)*4:
k:=1-I*n+n^2-I*n^3+I+I*n^2:
ck:=1+I*n+n^2+I*n^3-I-I*n^2:
m:=expand(k*ck):
Fix the results of the previous - (3,4) - step.
> f01:=a:H4:= 0: h4:= 0: g3:= 0: H101:= 0: h101:= 0: g001:= 0: G3:=
0: H02:= -2/3*r^4-14/3*r^4*n^2+4/3*r^4*n-27/2*r^4*n^4-
125/6*r^4*n^6+50/3*r^4*n^5-1/6*r^4*n^14+2/3*r^4*n^13-
9*r^4*n^10+40/3*r^4*n^9-13/6*r^4*n^12+14/3*r^4*n^11-
55/3*r^4*n^8+20*r^4*n^7+22/3*r^4*n^3: h02:=
-8*r^2*f01*n^9+2*r^2*n^10*f01-2/3*r^4+8*r^2*f01-
19/3*r^4*n^2+2*r^4*n-139/6*r^4*n^4-177/4*r^4*n^6+203/6*r^4*n^5-
23/12*r^4*n^14+31/6*r^4*n^13-1/12*r^4*n^16+1/2*r^4*n^15-
63/2*r^4*n^10+125/3*r^4*n^9-34/3*r^4*n^12+61/3*r^4*n^11-
195/4*r^4*n^8+295/6*r^4*n^7+38/3*r^4*n^3+22*r^2*f01*n^8-
16*r^2*f01*n-56*r^2*f01*n^3+40*r^2*f01*n^2+74*r^2*f01*n^4-
72*r^2*f01*n^5+62*n^6*r^2*f01-40*n^7*r^2*f01: g11:=
-5/6*r^3*n^6+r^3*n^5-3/2*r^3*n^4+r^3*n^3-7/6*r^3*n^2+1/3*r^3*n-
1/3*r^3-1/6*r^3*n^8+1/3*r^3*n^7: G11:= -4*r*f01*n-
4*r*f01*n^3+6*r*f01*n^2+1/6*r^3*n^9-1/2*r^3*n^8-
1/3*r^3+4*r*f01+2/3*r^3*n+13/6*r^3*n^3-3/2*r^3*n^2+5/2*r^3*n^5-
5/2*r^3*n^4+7/6*r^3*n^7-11/6*r^3*n^6+2*r*n^4*f01: f2:= 1/6*r^2*n^6-
1/3*r^2*n^5+1/2*r^2*n^4-5/12*r^2*n^3+1/2*r^2*n^2-1/6*r^2*n-
1/12*n^7*r^2+f01*n+1/6*r^2-f01: F01:= -1/12*r^2*n^6+1/6*r^2*n^5-
1/3*r^2*n^4+1/3*r^2*n^3-5/12*r^2*n^2+1/6*r^2*n-f01*n+f01-1/6*r^2:
h21:= -2/3*r^4-14/3*r^4*n^2+4/3*r^4*n-27/2*r^4*n^4-
125/6*r^4*n^6+50/3*r^4*n^5-1/6*r^4*n^14+2/3*r^4*n^13-
9*r^4*n^10+40/3*r^4*n^9-13/6*r^4*n^12+14/3*r^4*n^11-
55/3*r^4*n^8+20*r^4*n^7+22/3*r^4*n^3: F2:= -1/12*r^2*n^6-
1/6*r^2*n^4-1/12*r^2*n^2+f01: H21:= -8*r^2*f01*n^9+2*r^2*n^10*f01-
2/3*r^4+8*r^2*f01-6*r^4*n^2+2*r^4*n-125/6*r^4*n^4-
75/2*r^4*n^6+181/6*r^4*n^5-5/6*r^4*n^14+17/6*r^4*n^13+1/6*r^4*n^15-
67/3*r^4*n^10+95/3*r^4*n^9-41/6*r^4*n^12+41/3*r^4*n^11-
115/3*r^4*n^8+245/6*r^4*n^7+12*r^4*n^3+22*r^2*f01*n^8-16*r^2*f01*n-
56*r^2*f01*n^3+40*r^2*f01*n^2+74*r^2*f01*n^4-
72*r^2*f01*n^5+62*n^6*r^2*f01-40*n^7*r^2*f01: G001:=
2*r*f01*n+2*r*f01*n^3-3*r*f01*n^2+1/3*r^3*n^9-11/12*r^3*n^8-

```



$1/3*r^3-2*r*f01+2/3*r^3*n+7/3*r^3*n^3-5/3*r^3*n^2+3*r^3*n^5-37/12*r^3*n^4+5/3*r^3*n^7-31/12*r^3*n^6-r*n^4*f01-1/12*r^3*n^10:$

**Introduce new variables**

```
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
```

**Write a mapping**

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
```

e3:=subs ([z=f,cz=cf],2\*T3)+I\*(h-ch):

**Write the coefficients of the (4,5)-jet.**

```
> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):
```

```
P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):
```

```
Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
```

:

```
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):
```

```
Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):
```

**Write the equations.**

```
> EQ:={
```

```
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
```

$$P[1, 1, 1, 0]=0,$$

$$P1[1, 0, 0, 1]=0, P2[1, 0, 0, 1]=0,$$

$$Q1[5, 0, 0, 0]=0, Q2[5, 0, 0, 0]=0,$$

$$Q1[4, 1, 0, 0]=0, Q2[4, 1, 0, 0]=0,$$

$$Q1[3, 2, 0, 0]=0, Q2[3, 2, 0, 0]=0,$$

$$Q1[3, 0, 1, 0]=0, Q2[3, 0, 1, 0]=0,$$

$$Q1[2, 1, 1, 0]=0, Q2[2, 1, 1, 0]=0,$$

$$Q1[1, 0, 2, 0]=0, Q2[1, 0, 2, 0]=0,$$

$$Q1[2, 0, 0, 1]=0, Q2[2, 0, 0, 1]=0,$$

$$Q[1, 1, 0, 1]=0,$$

$$Q[0, 0, 1, 1]=0$$

}:

**Number of equations - 26.**

> sol:=solve(EQ):

> sol[5];

$$\{n = n, f001 = -\frac{1}{144}(-r^4 n^{12} + 4r^4 n^{11} - 13r^4 n^{10} + 26r^4 n^9 - 48r^4 n^8 + 64r^4 n^7$$

$$- 82r^4 n^6 + 76r^4 n^5 - 73r^4 n^4 + 44r^4 n^3 - 33r^4 n^2 + 10r^4 n - 6r^4 + 72a^2) / ($$

$$r(1 + n^2)), a = a, h12 = \frac{143}{18}r^5 n^4 - \frac{1}{12}r^5 n^{17} - 12r^3 n^{10} a - 32r n^5 a^2 - 16r a^2 n$$

$$+ 20r^3 n^3 a - \frac{5}{3}n^{12} r^3 a - 44r^3 n^6 a + 32r a^2 n^2 + 9r^5 n^{12} - \frac{40}{3}r^5 n^{11} + \frac{1}{72}n^{18} r^5$$

$$+ 42r n^4 a^2 - 8n^7 r a^2 + 20n^6 r a^2 - 40r n^3 a^2 + 8r a^2 - \frac{4}{3}r^3 a + \frac{124}{3}r^3 n^7 a$$

$$+ \frac{187}{72}r^5 n^{14} + 22r^3 n^9 a - \frac{19}{18}r^5 n^{15} + \frac{13}{36}r^5 n^{16} + \frac{16}{3}r^3 n^{11} a - \frac{5}{9}r^5 n + \frac{37}{18}r^5 n^2$$

$$- \frac{47}{4}r^5 n^5 + \frac{407}{24}r^5 n^6 - \frac{745}{36}r^5 n^9 + \frac{121}{3}r^3 n^5 a + \frac{1291}{72}r^5 n^{10} - \frac{359}{18}r^5 n^7 - \frac{32}{3}r^3 n^2 a$$

$$+ \frac{793}{36}r^5 n^8 - 31r^3 n^4 a - \frac{35}{9}r^5 n^3 + 4r^3 n a - \frac{185}{36}r^5 n^{13} - \frac{98}{3}r^3 a n^8 + \frac{2}{9}r^5$$

$$+ \frac{1}{3}n^{13} r^3 a + 2r n^8 a^2, r = r, H31 = \frac{34}{9}r^5 n^4 - \frac{1}{6}r^5 n^{17} - 24r^3 n^{10} a - 64r n^5 a^2$$

$$- 32r a^2 n + 40r^3 n^3 a - \frac{10}{3}n^{12} r^3 a - 88r^3 n^6 a + 64r a^2 n^2 + \frac{95}{9}r^5 n^{12} - \frac{44}{3}r^5 n^{11}$$

$$+ \frac{1}{36}n^{18} r^5 + 84r n^4 a^2 - 16n^7 r a^2 + 40n^6 r a^2 - 80r n^3 a^2 + 16r a^2 - \frac{8}{3}r^3 a$$

$$\begin{aligned}
& + \frac{248}{3} r^3 n^7 a + \frac{131}{36} r^5 n^{14} + 44 r^3 n^9 a - \frac{5}{3} r^5 n^{15} + \frac{11}{18} r^5 n^{16} + \frac{32}{3} r^3 n^{11} a - \frac{2}{9} r^5 n \\
& + \frac{5}{9} r^5 n^2 - \frac{15}{2} r^5 n^5 + \frac{397}{36} r^5 n^6 - \frac{115}{6} r^5 n^9 + \frac{242}{3} r^3 n^5 a + \frac{635}{36} r^5 n^{10} - \frac{139}{9} r^5 n^7 \\
& - \frac{64}{3} r^3 n^2 a + \frac{323}{18} r^5 n^8 - 62 r^3 n^4 a - 2 r^5 n^3 + 8 r^3 n a - \frac{121}{18} r^5 n^{13} - \frac{196}{3} r^3 a n^8 \\
& + \frac{2}{3} n^{13} r^3 a + 4 r n^8 a^2, G21 = \frac{10}{3} r^2 a n - \frac{37}{6} r^2 a n^2 - \frac{15}{2} r^2 a n^4 + \frac{17}{3} r^2 a n^5 \\
& - \frac{19}{6} n^6 r^2 a + \frac{7}{6} n^7 r^2 a + \frac{47}{6} r^2 a n^3 - \frac{1}{6} r^2 a n^8 + \frac{1}{18} r^4 + \frac{11}{18} r^4 n^2 - \frac{5}{18} r^4 n + \frac{143}{72} r^4 n^4 \\
& + \frac{53}{18} r^4 n^6 - \frac{67}{24} r^4 n^5 - \frac{1}{72} r^4 n^{13} + \frac{7}{9} r^4 n^{10} - \frac{53}{36} r^4 n^9 + \frac{7}{72} r^4 n^{12} - \frac{1}{3} r^4 n^{11} + \frac{79}{36} r^4 n^8 \\
& - \frac{17}{6} r^4 n^7 - \frac{25}{18} r^4 n^3 - \frac{5}{3} r^2 a + 2 a^2 n^2 - 4 a^2 n + 4 a^2, h31 = \frac{161}{18} r^5 n^4 - \frac{1}{18} r^5 n^{17} \\
& - 8 r^3 n^{10} a + 24 r^3 n^3 a - \frac{2}{3} n^{12} r^3 a - \frac{136}{3} r^3 n^6 a + \frac{175}{18} r^5 n^{12} - \frac{136}{9} r^5 n^{11} - \frac{8}{3} r^3 a \\
& + \frac{112}{3} r^3 n^7 a + \frac{23}{9} r^5 n^{14} + 16 r^3 n^9 a - r^5 n^{15} + \frac{5}{18} r^5 n^{16} + \frac{8}{3} r^3 n^{11} a - \frac{2}{3} r^5 n \\
& + \frac{20}{9} r^5 n^2 - \frac{253}{18} r^5 n^5 + \frac{175}{9} r^5 n^6 - \frac{145}{6} r^5 n^9 + \frac{128}{3} r^3 n^5 a + \frac{182}{9} r^5 n^{10} - \frac{71}{3} r^5 n^7 \\
& - 16 r^3 n^2 a + \frac{455}{18} r^5 n^8 - 38 r^3 n^4 a - \frac{14}{3} r^5 n^3 + \frac{16}{3} r^3 n a - \frac{11}{2} r^5 n^{13} - 28 r^3 a n^8 \\
& + \frac{2}{9} r^5, G4 = 0, h5 = 0, H5 = 0, h011 = 0, g4 = 0, g21 = r^2 a n - \frac{5}{2} r^2 a n^2 - 2 r^2 a n^4 \\
& + r^2 a n^5 - \frac{1}{2} n^6 r^2 a + 2 r^2 a n^3 + \frac{1}{6} r^4 + \frac{17}{18} r^4 n^2 - \frac{1}{3} r^4 n + \frac{17}{8} r^4 n^4 + \frac{43}{18} r^4 n^6 \\
& - \frac{22}{9} r^4 n^5 + \frac{1}{3} r^4 n^{10} - \frac{7}{9} r^4 n^9 + \frac{1}{72} r^4 n^{12} - \frac{1}{9} r^4 n^{11} + \frac{49}{36} r^4 n^8 - 2 r^4 n^7 - \frac{13}{9} r^4 n^3
\end{aligned}$$

$$-r^2 a, H12 = \frac{161}{36} r^5 n^4 - \frac{1}{36} r^5 n^{17} - 4 r^3 n^{10} a + 12 r^3 n^3 a - \frac{1}{3} n^{12} r^3 a - \frac{68}{3} r^3 n^6 a$$

$$+ \frac{175}{36} r^5 n^{12} - \frac{68}{9} r^5 n^{11} - \frac{4}{3} r^3 a + \frac{56}{3} r^3 n^7 a + \frac{23}{18} r^5 n^{14} + 8 r^3 n^9 a - \frac{1}{2} r^5 n^{15}$$

$$+ \frac{5}{36} r^5 n^{16} + \frac{4}{3} r^3 n^{11} a - \frac{1}{3} r^5 n + \frac{10}{9} r^5 n^2 - \frac{253}{36} r^5 n^5 + \frac{175}{18} r^5 n^6 - \frac{145}{12} r^5 n^9$$

$$+ \frac{64}{3} r^3 n^5 a + \frac{91}{9} r^5 n^{10} - \frac{71}{6} r^5 n^7 - 8 r^3 n^2 a + \frac{455}{36} r^5 n^8 - 19 r^3 n^4 a - \frac{7}{3} r^5 n^3$$

$$+ \frac{8}{3} r^3 n a - \frac{11}{4} r^5 n^{13} - 14 r^3 a n^8 + \frac{1}{9} r^5, h201 = \frac{1}{144} (-493 r^2 n^{10} + 344 r^2 n^{11}$$

$$- 186 r^2 n^{12} - 704 n^8 r^2 - 8 r^2 - 591 r^2 n^6 + 454 r^2 n^5 - 290 r^2 n^4 + 160 r^2 n^3$$

$$- 76 r^2 n^2 + 24 r^2 n + 708 n^7 r^2 - 192 a n + 96 a + 650 n^9 r^2 + 24 n^{12} a + 288 n^{10} a$$

$$- 1536 n^5 a + 1632 n^6 a + 1368 n^4 a + 576 a n^2 - 864 a n^3 - 1344 n^7 a - 576 n^9 a$$

$$- 96 n^{11} a + 1008 a n^8 + n^{18} r^2 - 2 r^2 n^{17} - 25 r^2 n^{14} + 90 r^2 n^{13} + 4 r^2 n^{16} + 4 r^2 n^{15})$$

$$r^3, F3 = \frac{1}{72} (r^4 n^{12} - r^4 n^{11} + 6 r^4 n^{10} - 6 r^4 n^9 + 14 r^4 n^8 - 14 r^4 n^7 + 16 r^4 n^6$$

$$- 12 n^6 r^2 a - 16 r^4 n^5 + 9 r^4 n^4 - 24 r^2 a n^4 - 9 r^4 n^3 + 2 r^4 n^2 - 12 r^2 a n^2 - 2 r^4 n$$

$$+ 72 a^2) / (r(1+n^2)), H201 = - \frac{1}{144} (-81 n^{18} r^4 + 17 n^{19} r^4 + 274 r^4 n^{17} - 2 r^4 n^{20}$$

$$+ 288 r^2 a n + 1584 r^2 a n^9 - 864 r^2 n^{10} a - 768 r^2 a n^2 - 2232 r^2 a n^4 + 2904 r^2 a n^5$$

$$- 3168 n^6 r^2 a + 2976 n^7 r^2 a + 1440 r^2 a n^3 - 2352 r^2 a n^8 - 40 r^4 - 516 r^4 n^2$$

$$+ 144 r^4 n - 2546 r^4 n^4 - 6749 r^4 n^6 + 4372 r^4 n^5 - 3091 r^4 n^{14} + 5104 r^4 n^{13}$$

$$- 11227 r^4 n^{10} + 11642 r^4 n^9 - 7456 r^4 n^{12} + 9659 r^4 n^{11} - 10886 r^4 n^8 + 9029 r^4 n^7$$

$$+ 1204 r^4 n^3 - 734 r^4 n^{16} + 1627 r^4 n^{15} - 96 r^2 a + 2304 a^2 n^2 - 1152 a^2 n + 576 a^2$$

$$+ 24 n^{13} r^2 a + 144 n^8 a^2 - 2304 n^5 a^2 - 120 n^{12} r^2 a + 3024 n^4 a^2 - 576 n^7 a^2$$

$$+ 1440 n^6 a^2 - 2880 n^3 a^2 + 384 r^2 n^{11} a) r, H011 = \frac{1}{72} (-493 r^2 n^{10} + 344 r^2 n^{11}$$

$$- 186 r^2 n^{12} - 704 n^8 r^2 - 8 r^2 - 591 r^2 n^6 + 454 r^2 n^5 - 290 r^2 n^4 + 160 r^2 n^3$$

$$\begin{aligned}
& -76 r^2 n^2 + 24 r^2 n + 708 n^7 r^2 - 192 a n + 96 a + 650 n^9 r^2 + 24 n^{12} a + 288 n^{10} a \\
& - 1536 n^5 a + 1632 n^6 a + 1368 n^4 a + 576 a n^2 - 864 a n^3 - 1344 n^7 a - 576 n^9 a \\
& - 96 n^{11} a + 1008 a n^8 + n^{18} r^2 - 2 r^2 n^{17} - 25 r^2 n^{14} + 90 r^2 n^{13} + 4 r^2 n^{16} + 4 r^2 n^{15})
\end{aligned}$$

$$\begin{aligned}
r^3, G02 = & -\frac{1}{3} r^2 a n + \frac{5}{6} r^2 a n^2 + \frac{2}{3} r^2 a n^4 - \frac{1}{3} r^2 a n^5 + \frac{1}{6} n^6 r^2 a - \frac{2}{3} r^2 a n^3 + \frac{1}{18} r^4 \\
& + \frac{19}{54} r^4 n^2 - \frac{1}{9} r^4 n + \frac{7}{8} r^4 n^4 + \frac{59}{54} r^4 n^6 - \frac{26}{27} r^4 n^5 + \frac{2}{9} r^4 n^{10} - \frac{11}{27} r^4 n^9 + \frac{5}{216} r^4 n^{12}
\end{aligned}$$

$$\begin{aligned}
& -\frac{2}{27} r^4 n^{11} + \frac{77}{108} r^4 n^8 - \frac{8}{9} r^4 n^7 - \frac{14}{27} r^4 n^3 + \frac{1}{3} r^2 a, f11 = \frac{5}{216} r^3 n^9 - \frac{11}{216} r^3 n^8 \\
& + \frac{1}{8} r^3 n^7 - \frac{13}{72} r^3 n^6 + \frac{17}{72} r^3 n^5 - \frac{17}{72} r^3 n^4 + \frac{41}{216} r^3 n^3 - \frac{1}{3} r a n^3 - \frac{29}{216} r^3 n^2 + \frac{1}{18} r^3 n
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3} r a n + \frac{1}{3} r a n^2 - \frac{1}{36} r^3 + \frac{1}{3} r a, G101 = -\frac{2}{3} r^2 a n + \frac{7}{6} r^2 a n^2 + \frac{4}{3} r^2 a n^4 - r^2 a n^5 \\
& + \frac{1}{2} n^6 r^2 a - \frac{1}{6} n^7 r^2 a - \frac{3}{2} r^2 a n^3 + \frac{5}{36} r^4 + \frac{13}{12} r^4 n^2 - \frac{13}{36} r^4 n + \frac{463}{144} r^4 n^4 + \frac{347}{72} r^4 n^6
\end{aligned}$$

$$\begin{aligned}
& -\frac{199}{48} r^4 n^5 + \frac{1}{72} r^4 n^{14} - \frac{13}{144} r^4 n^{13} + \frac{61}{36} r^4 n^{10} - \frac{67}{24} r^4 n^9 + \frac{47}{144} r^4 n^{12} - \frac{5}{6} r^4 n^{11} \\
& + \frac{283}{72} r^4 n^8 - \frac{167}{36} r^4 n^7 - \frac{23}{12} r^4 n^3 + \frac{1}{3} r^2 a - a^2 n^2 + 2 a^2 n - 2 a^2, f3 = \frac{1}{144} (-48 r^2 a n \\
& + 144 r^2 a n^2 + 144 r^2 a n^4 - 96 r^2 a n^5 + 48 n^6 r^2 a - 24 n^7 r^2 a - 120 r^2 a n^3 - 4 r^4
\end{aligned}$$

$$\begin{aligned}
& -36 r^4 n^2 + 12 r^4 n - 111 r^4 n^4 - 164 r^4 n^6 + 130 r^4 n^5 + 2 r^4 n^{13} - 48 r^4 n^{10} \\
& + 80 r^4 n^9 - 7 r^4 n^{12} + 22 r^4 n^{11} - 126 r^4 n^8 + 140 r^4 n^7 + 62 r^4 n^3 + 48 r^2 a + 144 a^2 n \\
& - 144 a^2) / (r(1+n^2)), g101 = \frac{1}{144} (r^2 n^{12} + 8 n^9 r^2 - 14 n^8 r^2 + 32 n^7 r^2 - 36 r^2 n^6
\end{aligned}$$

$$\begin{aligned}
& + 24 n^6 a + 48 r^2 n^5 - 48 n^5 a + 96 n^4 a - 39 r^2 n^4 - 96 a n^3 + 32 r^2 n^3 - 20 r^2 n^2 \\
& + 120 a n^2 + 8 r^2 n - 48 a n - 4 r^2 + 48 a) r^2, F001 = \frac{1}{288} (-2 r^4 n^{13} + 11 r^4 n^{12} \\
& - 34 r^4 n^{11} + 78 r^4 n^{10} - 140 r^4 n^9 + 210 r^4 n^8 - 260 r^4 n^7 + 280 r^4 n^6 + 24 n^6 r^2 a
\end{aligned}$$

$$-250 r^4 n^5 - 48 r^2 a n^5 + 195 r^4 n^4 + 96 r^2 a n^4 - 122 r^4 n^3 - 96 r^2 a n^3 + 66 r^4 n^2 + 120 r^2 a n^2 - 24 r^4 n - 48 r^2 a n + 144 a^2 n + 8 r^4 + 48 r^2 a - 144 a^2) / (r(1+n^2))$$

$$), g02 = -\frac{2}{3} r^2 a n + \frac{3}{2} r^2 a n^2 + \frac{13}{6} r^2 a n^4 - \frac{5}{3} r^2 a n^5 + \frac{7}{6} n^6 r^2 a - \frac{1}{2} n^7 r^2 a - \frac{11}{6} r^2 a n^3$$

$$+ \frac{1}{6} r^2 a n^8 + \frac{11}{36} r^4 + \frac{71}{36} r^4 n^2 - \frac{11}{18} r^4 n + \frac{743}{144} r^4 n^4 + \frac{1015}{144} r^4 n^6 - \frac{55}{9} r^4 n^5 + \frac{1}{48} r^4 n^{14}$$

$$- \frac{1}{9} r^4 n^{13} + \frac{157}{72} r^4 n^{10} - \frac{65}{18} r^4 n^9 + \frac{59}{144} r^4 n^{12} - \frac{37}{36} r^4 n^{11} + \frac{385}{72} r^4 n^8 - \frac{115}{18} r^4 n^7$$

$$- \frac{109}{36} r^4 n^3 + \frac{1}{3} r^2 a - 4 a^2 n^2 + 8 a^2 n - 8 a^2, CI = \frac{1}{36} (-r^4 n^{12} + 3 r^4 n^{11} - 10 r^4 n^{10}$$

$$+ 18 r^4 n^9 - 36 r^4 n^8 + 42 r^4 n^7 - 18 n^6 r^2 a - 64 r^4 n^6 + 48 r^4 n^5 + 36 r^2 a n^5$$

$$- 72 r^2 a n^4 - 61 r^4 n^4 + 72 r^2 a n^3 + 27 r^4 n^3 - 90 r^2 a n^2 - 30 r^4 n^2 + 6 r^4 n$$

$$+ 36 r^2 a n + 216 a^2 - 6 r^4 - 36 r^2 a) / (r^2 (1+n^2)^2), F11 = \frac{5}{216} r^3 n^8 - \frac{1}{36} r^3 n^7$$

$$+ \frac{7}{72} r^3 n^6 - \frac{1}{12} r^3 n^5 + \frac{11}{72} r^3 n^4 - \frac{1}{12} r^3 n^3 + \frac{23}{216} r^3 n^2 - \frac{1}{3} r a n^2 - \frac{1}{36} r^3 n - \frac{1}{3} r a$$

$$+ \frac{1}{36} r^3, C2 = \frac{1}{108} r^2 n^6 + \frac{1}{54} r^2 n^4 + \frac{1}{108} r^2 n^2 \}$$

$$> CC2 := 1/108*r^2*n^6+1/54*r^2*n^4+1/108*r^2*n^2;$$

$$CC2 := \frac{1}{108} r^2 n^6 + \frac{1}{54} r^2 n^4 + \frac{1}{108} r^2 n^2$$

Then we see that ifgamma is non-zero, then it is not the cubic.

### Type IV for delta=0

>

First step: (3,4)-normalization -----  
 (2,3,4) jet

```
> restart: readlib(mttaylor):
A:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(y):
T:=exp(x):
```

```

S1:=mtaylor(4*S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=(4*T2-S2)*4:

```

**Result check.**

```

> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));

```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 1 + I$$

$$T_{3,0,0,0} := 0$$

**Introduce the notations.**

```
> k:=1+I: ck:=1-I: m:=2:
```

**Introduce new variables**

```

> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):

```

**Write a mapping**

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:

```

```

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:

```



```
g:=
r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (3,4)-jet.**

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
```

```

Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**17 equations    Solve the system.**

```

> sol:=solve(EQ):
> sol[2];
{G001 = 2 r f2 - 2/3 r^3, F01 = -f2, F2 = -f2 + 1/6 r^2, f01 = -f2 + 1/6 r^2, f2 = f2, r = r,
H02 = - 2/3 r^4, G11 = -4 r f2 + 1/3 r^3, g11 = - 1/3 r^3, h21 = - 2/3 r^4, H21 = -8 r^2 f2 + 2/3 r^4,
h02 = -8 r^2 f2 + 2/3 r^4, H101 = 0, h101 = 0, h4 = 0, g3 = 0, g001 = 0, G3 = 0, H4 = 0}

```

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```

> restart:readlib(mtaylor):
A:=0:B:=0:x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(y):
T:=exp(x):
S1:=mtaylor(4*S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=(4*T2-S2)*4:
k:=1+I: ck:=1-I: m:=2:

```

**Fix the results of the previous - (3,4) - step.**

```

> f2:=a: F2:=-f2+1/6*r^2: F01:= -f2: f01:= -f2+1/6*r^2: H02:=
-2/3*r^4: G11:= -4*r*f2+1/3*r^3: h21:= -2/3*r^4: h02:=
-8*r^2*f2+2/3*r^4: G001:= 2*r*f2-2/3*r^3: H21:= -8*r^2*f2+2/3*r^4:
g11:= -1/3*r^3: g001:= 0: G3:= 0: H101:= 0: h101:= 0: H4:= 0: h4:=

```

```
0: g3:= 0:
```

### Introduce new variables

```
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):  
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-I  
I*C2)*cz^3*z^2):  
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2+(C1+I*C2)*z^3*cz^2+(C1-I  
I*C2)*cz^3*z^2):
```

### Write a mapping

```
> f:=  
r*ck*z+  
(F2+I*f2)*z^2+(F01+I*f01)*w2+  
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:
```

```
cf:=  
r*k*cz+  
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+  
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:
```

```
g:=  
r^2*m*w2+  
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+  
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+  
(G101+I*g101)*z*w3:
```

```
cg:=  
r^2*m*cw2+  
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+  
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-I  
I*g101)*cz*cw3:
```

```
h:=  
r^3*m^2*w3+  
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+  
(H101+I*h101)*z*w3+  
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+  
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:
```

```
ch:=  
r^3*m^2*cw3+  
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-I  
I*h101)*cz*cw3+  
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-I  
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:
```

### Write the relations for the mapping.

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
```

e3:=subs ([z=f,cz=cf],2\*T3)+I\*(h-ch):

**Write the coefficients of the (4,5)-jet.**

```
> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):
```

```
P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):
```

```
Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):
```

```
Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):
```

**Write the equations.**

```
> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
```

$P[1, 1, 1, 0]=0,$   
 $P1[1, 0, 0, 1]=0, P2[1, 0, 0, 1]=0,$   
  
 $Q1[5, 0, 0, 0]=0, Q2[5, 0, 0, 0]=0,$   
 $Q1[4, 1, 0, 0]=0, Q2[4, 1, 0, 0]=0,$   
 $Q1[3, 2, 0, 0]=0, Q2[3, 2, 0, 0]=0,$   
 $Q1[3, 0, 1, 0]=0, Q2[3, 0, 1, 0]=0,$   
 $Q1[2, 1, 1, 0]=0, Q2[2, 1, 1, 0]=0,$   
 $Q1[1, 0, 2, 0]=0, Q2[1, 0, 2, 0]=0,$   
 $Q1[2, 0, 0, 1]=0, Q2[2, 0, 0, 1]=0,$   
 $Q[1, 1, 0, 1]=0,$   
 $Q[0, 0, 1, 1]=0$   
 $\}$ :

**Number of equations - 26.**

> sol:=solve(EQ):

> sol[2];

$$\{a = a, r = r, g21 = r^2 a, h31 = \frac{8}{3} r^3 a - \frac{2}{9} r^5, f3 = -\frac{a^2}{r}, H31 = 16 r a^2 - \frac{8}{3} r^3 a,$$

$$f001 = -\frac{1}{36} \frac{-6 r^2 a + 18 a^2 - r^4}{r}, CI = \frac{1}{6} \frac{-6 r^2 a + 36 a^2 - r^4}{r^2},$$

$$F3 = \frac{1}{36} \frac{36 a^2 + r^4 - 12 r^2 a}{r}, f11 = -\frac{1}{3} r a + \frac{1}{36} r^3, F001 = -\frac{1}{24} \frac{12 a^2 - r^4}{r},$$

$$F11 = \frac{1}{3} r a - \frac{1}{36} r^3, g02 = \frac{7}{3} r^2 a + \frac{5}{36} r^4 - 8 a^2, h12 = \frac{2}{9} r^5 + 8 r a^2 - \frac{4}{3} r^3 a,$$

$$H12 = \frac{4}{3} r^3 a - \frac{1}{9} r^5, G02 = -\frac{1}{3} r^2 a + \frac{1}{9} r^4, h201 = \frac{1}{18} r^5 - \frac{2}{3} r^3 a,$$

$$H201 = \frac{2}{3} r^3 a - 4 r a^2 + \frac{5}{18} r^5, G101 = \frac{1}{3} r^2 a - 2 a^2 + \frac{5}{36} r^4, H011 = -\frac{4}{3} r^3 a + \frac{1}{9} r^5,$$

$$g101 = -\frac{1}{3} r^2 a + \frac{1}{36} r^4, C2 = 0, G21 = 4 a^2 + \frac{1}{3} r^2 a - \frac{1}{9} r^4, h011 = 0, H5 = 0, h5 = 0,$$

$$G4 = 0, g4 = 0\}$$

> CC1:= 1/6\*(-6\*r^2\*a-r^4+36\*a^2)/(r^2);

$$CC1 := \frac{1}{6} \frac{-6 r^2 a + 36 a^2 - r^4}{r^2}$$

> solve(CC1=0, a);

$$\frac{1}{6} \left( \frac{1}{2} + \frac{1}{2} \sqrt{5} \right) r^2, \frac{1}{6} \left( \frac{1}{2} - \frac{1}{2} \sqrt{5} \right) r^2$$

>

## Third Step: (5,6)-normalization ----- (4,5,6) jet

```
> restart:readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
a:=p*r^2: A:=0: B:=0:J:=0:
C1:=0:
C2:=0:
S:=exp(y):
T:=exp(x):
S1:=mtaylor(4*S,[z,cz],8): T1:=mtaylor(T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=(4*T2-S2)*4:
k:=1+I: ck:=1-I: m:=2:
> f2:=a: F2:=-f2+1/6*r^2: F01:= -f2: f01:= -f2+1/6*r^2: H02:=
-2/3*r^4: G11:= -4*r*f2+1/3*r^3: h21:= -2/3*r^4: h02:=
-8*r^2*f2+2/3*r^4: G001:= 2*r*f2-2/3*r^3: H21:= -8*r^2*f2+2/3*r^4:
g11:= -1/3*r^3: g001:= 0: G3:= 0: H101:= 0: h101:= 0: H4:= 0: h4:=
0: g3:= 0:
G21:= 4*a^2+1/3*r^2*a-1/9*r^4: H12:= -1/9*r^5+4/3*r^3*a: g21:=
r^2*a: h31:= 8/3*r^3*a-2/9*r^5: h12:= -4/3*r^3*a+2/9*r^5+8*r*a^2:
H31:= 16*r*a^2-8/3*r^3*a: f3:= -a^2/r: F3:= 1/36*(r^4+36*a^2-
12*r^2*a)/r: f11:= -1/3*r*a+1/36*r^3: F001:= -1/24*(-r^4+12*a^2)/r:
f001:= -1/36*(-6*r^2*a-r^4+18*a^2)/r: g02:= 7/3*r^2*a+5/36*r^4-
8*a^2: F11:= 1/3*r*a-1/36*r^3: G02:= -1/3*r^2*a+1/9*r^4: h201:=
1/18*r^5-2/3*r^3*a: H201:= 2/3*r^3*a+5/18*r^5-4*r*a^2: H011:=
1/9*r^5-4/3*r^3*a: G101:= 1/3*r^2*a+5/36*r^4-2*a^2: g101:=
-1/3*r^2*a+1/36*r^4: H5:= 0: h011:= 0: h5:= 0: g4:= 0: G4:= 0:
```

### Write a mapping

```
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3+
(F4+I*f4)*z^4+(F21+I*f21)*z^2*w2+(F02+I*f02)*w2^2+
(F101+I*f101)*z*w3:
cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3+
(F4-I*f4)*cz^4+(F21-I*f21)*cz^2*cw2+(F02-I*f02)*cw2^2+(F101-
I*f101)*cz*cw3:
g:=
```

```

r^2*m*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3+
(G5+I*g5)*z^5+(G31+I*g31)*z^3*w2+(G12+I*g12)*z*w2^2+
(G201+I*g201)*z^2*w3+(G011+I*g011)*w2*w3:

```

```

cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3+
(G5-I*g5)*cz^5+(G31-I*g31)*cz^3*cw2+(G12-I*g12)*cz*cw2^2+(G201-
I*g201)*cz^2*cw3+(G011-I*g011)*cw2*cw3:

```

```

h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3+
(H6+I*h6)*z^6+(H41+I*h41)*z^4*w2+(H22+I*h22)*z^2*w2^2+
(H03+I*h03)*w2^3+(H301+I*h301)*z^3*w3+(H111+I*h111)*z*w2*m^2*w3+
(H002+I*h002)*w3^2:

```

```

ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3+
(H6-I*h6)*cz^6+(H41-I*h41)*cz^4*cw2+(H22-I*h22)*cz^2*cw2^2+(H03-
I*h03)*cw2^3+(H301-I*h301)*cz^3*cw3+(H111-I*h111)*cz*cw2*cw3+(H002-
I*h002)*cw3^2:

```

### Introduce new variables

```

> w2:=u2+I*(z*cw+B*z^2*cw^2+(D1+I*D2)*z^4*cw+(E1+I*E2)*z^3*cw^2+
(E1-I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
cw2:=u2-I*(z*cw+B*z^2*cw^2+(D1+I*D2)*z^4*cw+(E1+I*E2)*z^3*cw^2+(E1-
I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
w3:=u3+I*(z^2*cw+cz^2*z+A*z^2*cw^2+(C1+I*C2)*z^3*cw^2+(C1-
I*C2)*cz^3*z^2+J*z^3*cw^3):
cw3:=u3-I*(z^2*cw+cz^2*z+A*z^2*cw^2+(C1+I*C2)*z^3*cw^2+(C1-
I*C2)*cz^3*z^2+J*z^3*cw^3):

```

### Write the relations for the mapping.

```

> e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):

```

## Write the coefficients of the (5,6)-jet.

>

```
P[5,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,z)))):
P[4,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,cz)))):
P[3,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,cz,cz)))):
P[3,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,u2)))):
P[2,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,u2)))):
P[1,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2,u2)))):
P[2,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u3)))):
P[1,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u3)))):
P[0,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u2,u3)))):
```

```
P1[5,0,0,0]:=subs(I=0,P[5,0,0,0]):
P2[5,0,0,0]:=normal(expand((P[5,0,0,0]-P1[5,0,0,0])*I)):
P1[4,1,0,0]:=subs(I=0,P[4,1,0,0]):
P2[4,1,0,0]:=normal(expand((P[4,1,0,0]-P1[4,1,0,0])*I)):
P1[3,2,0,0]:=subs(I=0,P[3,2,0,0]):
P2[3,2,0,0]:=normal(expand((P[3,2,0,0]-P1[3,2,0,0])*I)):
P1[3,0,1,0]:=subs(I=0,P[3,0,1,0]):
P2[3,0,1,0]:=normal(expand((P[3,0,1,0]-P1[3,0,1,0])*I)):
P1[2,1,1,0]:=subs(I=0,P[2,1,1,0]):
P2[2,1,1,0]:=normal(expand((P[2,1,1,0]-P1[2,1,1,0])*I)):
P1[1,0,2,0]:=subs(I=0,P[1,0,2,0]):
P2[1,0,2,0]:=normal(expand((P[1,0,2,0]-P1[1,0,2,0])*I)):
P1[2,0,0,1]:=subs(I=0,P[2,0,0,1]):
P2[2,0,0,1]:=normal(expand((P[2,0,0,1]-P1[2,0,0,1])*I)):
```

```
Q[6,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z,z,z)))):
Q[5,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z,cz)))):
Q[4,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,cz,cz)))):
Q[3,3,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz,cz,cz)))):
Q[4,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,u2)))):
Q[3,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz,u2)))):
```



```

Q[2,2,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz,u2)))):
Q[2,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2,u2)))):
Q[1,1,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2,u2)))):
Q[0,0,3,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u2,u2)))):
Q[3,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u3)))):
Q[2,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u3)))):
Q[1,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u3)))):
Q[0,0,0,2]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u3,u3)))):

```

```

Q1[6,0,0,0]:=subs(I=0,Q[6,0,0,0]):
Q2[6,0,0,0]:=normal(expand((Q[6,0,0,0]-Q1[6,0,0,0])*I)):
Q1[5,1,0,0]:=subs(I=0,Q[5,1,0,0]):
Q2[5,1,0,0]:=normal(expand((Q[5,1,0,0]-Q1[5,1,0,0])*I)):
Q1[4,2,0,0]:=subs(I=0,Q[4,2,0,0]):
Q2[4,2,0,0]:=normal(expand((Q[4,2,0,0]-Q1[4,2,0,0])*I)):

```

```

Q1[4,0,1,0]:=subs(I=0,Q[4,0,1,0]):
Q2[4,0,1,0]:=normal(expand((Q[4,0,1,0]-Q1[4,0,1,0])*I)):
Q1[3,1,1,0]:=subs(I=0,Q[3,1,1,0]):
Q2[3,1,1,0]:=normal(expand((Q[3,1,1,0]-Q1[3,1,1,0])*I)):

```

```

Q1[2,0,2,0]:=subs(I=0,Q[2,0,2,0]):
Q2[2,0,2,0]:=normal(expand((Q[2,0,2,0]-Q1[2,0,2,0])*I)):

```

```

Q1[3,0,0,1]:=subs(I=0,Q[3,0,0,1]):
Q2[3,0,0,1]:=normal(expand((Q[3,0,0,1]-Q1[3,0,0,1])*I)):
Q1[2,1,0,1]:=subs(I=0,Q[2,1,0,1]):
Q2[2,1,0,1]:=normal(expand((Q[2,1,0,1]-Q1[2,1,0,1])*I)):
Q1[1,0,1,1]:=subs(I=0,Q[1,0,1,1]):
Q2[1,0,1,1]:=normal(expand((Q[1,0,1,1]-Q1[1,0,1,1])*I)):

```

```
> EQ:={
```

```

P1[5,0,0,0]=0, P2[5,0,0,0]=0,
P1[4,1,0,0]=0, P2[4,1,0,0]=0,
P1[3,2,0,0]=0, P2[3,2,0,0]=0,
P1[3,0,1,0]=0, P2[3,0,1,0]=0,
P1[2,1,1,0]=0, P2[2,1,1,0]=0,
P1[1,0,2,0]=0, P2[1,0,2,0]=0,

```

```
P1[2,0,0,1]=0, P2[2,0,0,1]=0,
P[1,1,0,1]=0,
P[0,0,1,1]=0,
```

```
Q1[6,0,0,0]=0, Q2[6,0,0,0]=0,
Q1[5,1,0,0]=0, Q2[5,1,0,0]=0,
Q1[4,2,0,0]=0, Q2[4,2,0,0]=0,
Q[3,3,0,0]=0,
Q1[4,0,1,0]=0, Q2[4,0,1,0]=0,
Q1[3,1,1,0]=0, Q2[3,1,1,0]=0,
Q[2,2,1,0]=0,
Q1[2,0,2,0]=0, Q2[2,0,2,0]=0,
Q[1,1,2,0]=0,
Q[0,0,3,0]=0,
Q1[3,0,0,1]=0, Q2[3,0,0,1]=0,
Q1[2,1,0,1]=0, Q2[2,1,0,1]=0,
Q1[1,0,1,1]=0, Q2[1,0,1,1]=0,
Q[0,0,0,2]=0
}:
```

```
> sol:=solve(EQ);
sol := {h6 = 0, H6 = 0, h002 = 0, G5 = 0, g5 = 0, g011 = 0, r = 0, H22 = 0, h301 = 0,
G31 = 0, G011 = 0, H301 = 0, h03 = 0, h41 = 0, H41 = 0, H002 = 0, p = p, E2 = E2,
E1 = E1, D2 = D2, D1 = D1, F4 = F4, f4 = f4, F21 = F21, f21 = f21, F02 = F02,
f02 = f02, F101 = F101, f101 = f101, g12 = 0, G12 = 0, g31 = 0, H03 = 0, h22 = 0,
h111 = 0, H111 = 0, G201 = 0, g201 = 0}
```

>

There is no solution, So it is not the cubic.

## Type Va for non-zero alpha.

We put m:=alpha

First step: (3,4)-normalization -----

(2,3,4) jet

```
> restart:readlib(mttaylor):
A:=0:x:=(I*z-I*cz)/2:y:=(I*z+I*cz)/(2*I):
```

```

S:=exp(x+m*y)*sin(n*y):T:=exp(x+m*y)*cos(n*y):
S1:=mtaylor(2*S/m/n,[z,cz],8):T1:=mtaylor(16*T,[z,cz],9):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=-2*m*(T2+(4*n^2-4-4*m^2)*S2):

```

**Result check.**

```

>S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=factor(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))/2);
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));

```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := (n^2 + 2n + 1 + m^2)(n^2 - 2n + 1 + m^2)$$

$$T_{3,0,0,0} := 0$$

**Introduce the notations.**

```

>k:=(m^2+1-2*n+n^2)*(m^2+1+2*n+n^2):
ck:=(m^2+1-2*n+n^2)*(m^2+1+2*n+n^2):
M:=k*ck:

```

**Introduce new variables**

```

>w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+A*z^2*cz^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z^2*cz^2):

```

**Write a mapping**

```

>f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2:

```

```
cf:=
```

```
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:
```

```
g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (3,4)-jet.**

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
```

```

Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):

```

**Write 17 equations.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

```

```

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**Solve the system.**

```

> sol:=solve(EQ):

```

```

> sol[2]:

```

$$\begin{aligned}
\{r = r, h2l = & \frac{1}{6}r^4 + \frac{1}{6}m^{20}r^4 + \frac{5}{3}m^{18}r^4 + \frac{15}{2}m^{16}r^4 + 35m^{12}r^4 + \frac{5}{3}m^2r^4 - 20n^{14}r^4 \\
& - \frac{5}{3}n^2r^4 + \frac{15}{2}m^4r^4 + 35m^8r^4 + 20m^6r^4 + 42m^{10}r^4 - 20n^6r^4 + 35n^8r^4 - \frac{5}{3}n^{18}r^4 \\
& + \frac{15}{2}n^{16}r^4 + 35n^{12}r^4 + 20m^{14}r^4 + 20m^6n^{10}r^4 + \frac{140}{3}m^{12}n^2r^4 + \frac{70}{3}m^2n^{10}r^4 \\
& - \frac{70}{3}m^8n^{10}r^4 - \frac{140}{3}m^6n^{12}r^4 - \frac{100}{3}m^4n^{14}r^4 - \frac{35}{3}m^2n^{16}r^4 + 50m^{12}n^4r^4 \\
& + \frac{35}{3}m^{16}n^2r^4 + \frac{100}{3}m^{14}n^4r^4 + \frac{70}{3}m^{10}n^8r^4 + \frac{140}{3}m^{12}n^6r^4 + \frac{5}{3}n^{18}m^2r^4 \\
& + \frac{100}{3}m^{14}n^2r^4 + 35n^8m^{12}r^4 + \frac{5}{3}n^2m^{18}r^4 + \frac{15}{2}n^4m^{16}r^4 - \frac{140}{3}m^2n^{12}r^4 \\
& + 20n^6m^{14}r^4 + 42n^{10}m^{10}r^4 + 35n^{12}m^8r^4 + 20n^{14}m^6r^4 + \frac{15}{2}n^{16}m^4r^4 \\
& - \frac{100}{3}m^4n^2r^4 + \frac{100}{3}m^2n^4r^4 + \frac{20}{3}m^6n^8r^4 - \frac{140}{3}m^2n^6r^4 - 42n^{10}r^4 + \frac{15}{2}n^4r^4
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{6} n^{20} r^4 - 20 m^4 n^6 r^4 - \frac{35}{3} m^2 n^2 r^4 + 50 m^4 n^4 r^4 - \frac{25}{3} m^8 n^4 r^4 + \frac{20}{3} m^6 n^6 r^4 \\
& + \frac{70}{3} m^2 n^8 r^4 - \frac{140}{3} m^6 n^2 r^4 - \frac{70}{3} m^8 n^2 r^4 + 20 m^6 n^4 r^4 - 20 m^4 n^{10} r^4 - \frac{25}{3} m^4 n^8 r^4 \\
& + 50 m^4 n^{12} r^4 + \frac{100}{3} m^2 n^{14} r^4 - \frac{20}{3} m^8 n^6 r^4 + \frac{70}{3} m^{10} n^2 r^4 + 20 m^{10} n^6 r^4 \\
& - \frac{25}{3} m^8 n^8 r^4 + 20 m^{10} n^4 r^4, f2 = -\frac{1}{12} n^8 r^2 + \frac{1}{3} n^6 r^2 - \frac{1}{3} m^2 n^6 r^2 + \frac{1}{3} m^2 n^4 r^2 \\
& - \frac{1}{2} m^4 n^4 r^2 - \frac{1}{2} n^4 r^2 + \frac{1}{3} n^2 r^2 - \frac{1}{3} m^6 n^2 r^2 + \frac{1}{3} m^2 n^2 r^2 - \frac{1}{3} m^4 n^2 r^2 - \frac{1}{12} r^2 - \frac{1}{3} m^2 r^2 \\
& - \frac{1}{2} m^4 r^2 - \frac{1}{3} m^6 r^2 - \frac{1}{12} m^8 r^2, h4 = 0, g3 = 0, G3 = 0, g001 = 0, h101 = 0, H101 = 0, \\
& H4 = 0, f01 = f01, n = n, m = m,
\end{aligned}$$

$$\begin{aligned}
G11 & = 2 r f01 - 4 n^2 r f01 + 4 m^2 r f01 + 2 n^4 r f01 + 4 m^2 n^2 r f01 + 2 m^4 r f01, g11 = \\
& m^2 n^{10} r^3 + \frac{5}{2} m^8 n^4 r^3 + 3 m^8 n^2 r^3 + 2 m^2 n^4 r^3 - 3 m^2 n^2 r^3 - 3 m^2 n^8 r^3 + 2 m^6 n^4 r^3 \\
& - 2 m^4 n^6 r^3 + \frac{5}{2} m^4 n^8 r^3 + 2 m^6 n^2 r^3 + m^{10} n^2 r^3 - m^4 n^4 r^3 - 2 m^4 n^2 r^3 + \frac{1}{6} r^3 \\
& + \frac{1}{6} n^{12} r^3 + \frac{5}{2} m^8 r^3 + \frac{1}{6} m^{12} r^3 + m^{10} r^3 - \frac{10}{3} n^6 r^3 + \frac{10}{3} m^6 r^3 + \frac{5}{2} m^4 r^3 + \frac{5}{2} n^4 r^3 \\
& - n^{10} r^3 + m^2 r^3 - n^2 r^3 + \frac{5}{2} n^8 r^3 + 2 m^2 n^6 r^3 + \frac{10}{3} m^6 n^6 r^3, H21 = 30 m^8 r^2 f01 \\
& + 12 m^{10} r^2 f01 - 24 m^4 n^2 r^2 f01 + 36 m^8 n^2 r^2 f01 - 36 m^2 n^8 r^2 f01 + 30 m^4 r^2 f01 \\
& - 12 n^2 r^2 f01 + 24 m^2 n^4 r^2 f01 - 36 m^2 n^2 r^2 f01 + 24 m^2 n^6 r^2 f01 - 24 m^4 n^6 r^2 f01 \\
& - 12 m^4 n^4 r^2 f01 + 24 m^6 n^4 r^2 f01 + 24 m^6 n^2 r^2 f01 - 12 n^{10} r^2 f01 + 30 n^8 r^2 f01 \\
& - 40 n^6 r^2 f01 + 30 n^4 r^2 f01 + 2 m^{12} r^2 f01 + 2 r^2 f01 + 12 n^2 m^{10} r^2 f01 \\
& + 30 n^4 m^8 r^2 f01 + 12 n^{10} m^2 r^2 f01 + 30 n^8 m^4 r^2 f01 + 40 n^6 m^6 r^2 f01 + 2 n^{12} r^2 f01
\end{aligned}$$

$$\begin{aligned}
& + 40 m^6 r^2 f01 + 12 m^2 r^2 f01, H02 = \frac{1}{6} r^4 + \frac{1}{6} m^{20} r^4 + \frac{5}{3} m^{18} r^4 + \frac{15}{2} m^{16} r^4 + 35 m^{12} r^4 \\
& + \frac{5}{3} m^2 r^4 - 20 n^{14} r^4 - \frac{5}{3} n^2 r^4 + \frac{15}{2} m^4 r^4 + 35 m^8 r^4 + 20 m^6 r^4 + 42 m^{10} r^4 - 20 n^6 r^4 \\
& + 35 n^8 r^4 - \frac{5}{3} n^{18} r^4 + \frac{15}{2} n^{16} r^4 + 35 n^{12} r^4 + 20 m^{14} r^4 + 20 m^6 n^{10} r^4 + \frac{140}{3} m^{12} n^2 r^4 \\
& + \frac{70}{3} m^2 n^{10} r^4 - \frac{70}{3} m^8 n^{10} r^4 - \frac{140}{3} m^6 n^{12} r^4 - \frac{100}{3} m^4 n^{14} r^4 - \frac{35}{3} m^2 n^{16} r^4 \\
& + 50 m^{12} n^4 r^4 + \frac{35}{3} m^{16} n^2 r^4 + \frac{100}{3} m^{14} n^4 r^4 + \frac{70}{3} m^{10} n^8 r^4 + \frac{140}{3} m^{12} n^6 r^4 \\
& + \frac{5}{3} n^{18} m^2 r^4 + \frac{100}{3} m^{14} n^2 r^4 + 35 n^8 m^{12} r^4 + \frac{5}{3} n^2 m^{18} r^4 + \frac{15}{2} n^4 m^{16} r^4 \\
& - \frac{140}{3} m^2 n^{12} r^4 + 20 n^6 m^{14} r^4 + 42 n^{10} m^{10} r^4 + 35 n^{12} m^8 r^4 + 20 n^{14} m^6 r^4 \\
& + \frac{15}{2} n^{16} m^4 r^4 - \frac{100}{3} m^4 n^2 r^4 + \frac{100}{3} m^2 n^4 r^4 + \frac{20}{3} m^6 n^8 r^4 - \frac{140}{3} m^2 n^6 r^4 - 42 n^{10} r^4 \\
& + \frac{15}{2} n^4 r^4 + \frac{1}{6} n^{20} r^4 - 20 m^4 n^6 r^4 - \frac{35}{3} m^2 n^2 r^4 + 50 m^4 n^4 r^4 - \frac{25}{3} m^8 n^4 r^4 \\
& + \frac{20}{3} m^6 n^6 r^4 + \frac{70}{3} m^2 n^8 r^4 - \frac{140}{3} m^6 n^2 r^4 - \frac{70}{3} m^8 n^2 r^4 + 20 m^6 n^4 r^4 - 20 m^4 n^{10} r^4 \\
& - \frac{25}{3} m^4 n^8 r^4 + 50 m^4 n^{12} r^4 + \frac{100}{3} m^2 n^{14} r^4 - \frac{20}{3} m^8 n^6 r^4 + \frac{70}{3} m^{10} n^2 r^4 \\
& + 20 m^{10} n^6 r^4 - \frac{25}{3} m^8 n^8 r^4 + 20 m^{10} n^4 r^4, F2 = -\frac{1}{6} m n^8 r^2 - \frac{2}{3} m^3 n^6 r^2 + \frac{2}{3} m n^6 r^2 \\
& - m^5 n^4 r^2 - m n^4 r^2 + \frac{2}{3} m^3 n^4 r^2 + \frac{2}{3} m n^2 r^2 + \frac{2}{3} m^3 n^2 r^2 - \frac{2}{3} m^5 n^2 r^2 - \frac{2}{3} m^7 n^2 r^2 \\
& - \frac{2}{3} m^3 r^2 - m^5 r^2 - \frac{2}{3} m^7 r^2 - \frac{1}{6} m^9 r^2 - \frac{1}{6} m r^2 + f01, h02 = \frac{5}{3} m n^2 r^4 + 20 m n^6 r^4
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{6} m n^{20} r^4 + 42 m n^{10} r^4 + \frac{70}{3} m^9 n^2 r^4 - 35 m n^8 r^4 - \frac{15}{2} m n^{16} r^4 + 30 m^8 r^2 f01 \\
& + 20 m n^{14} r^4 + 12 m^{10} r^2 f01 - 24 m^4 n^2 r^2 f01 + 36 m^8 n^2 r^2 f01 - 36 m^2 n^8 r^2 f01 \\
& - \frac{100}{3} m^3 n^4 r^4 + 30 m^4 r^2 f01 - \frac{5}{3} n^{18} m^3 r^4 + 20 m^5 n^6 r^4 - 20 m^7 r^4 - \frac{15}{2} m^5 r^4 \\
& - 42 m^{11} r^4 - 35 m^9 r^4 - \frac{5}{3} m^3 r^4 - 35 m^{13} r^4 - 20 m^{15} r^4 - \frac{15}{2} m^{17} r^4 - \frac{5}{3} m^{19} r^4 \\
& - \frac{15}{2} m n^4 r^4 - 12 n^2 r^2 f01 + 24 m^2 n^4 r^2 f01 - 36 m^2 n^2 r^2 f01 + 24 m^2 n^6 r^2 f01 \\
& - 24 m^4 n^6 r^2 f01 - 12 m^4 n^4 r^2 f01 + 24 m^6 n^4 r^2 f01 + 24 m^6 n^2 r^2 f01 - 12 n^{10} r^2 f01 \\
& + 30 n^8 r^2 f01 - 40 n^6 r^2 f01 + 30 n^4 r^2 f01 - 35 n^{12} m^9 r^4 + 2 m^{12} r^2 f01 - 50 m^5 n^4 r^4 \\
& + 2 r^2 f01 - 35 m n^{12} r^4 + \frac{5}{3} m n^{18} r^4 + 12 n^2 m^{10} r^2 f01 + 30 n^4 m^8 r^2 f01 \\
& + 12 n^{10} m^2 r^2 f01 + 30 n^8 m^4 r^2 f01 + 40 n^6 m^6 r^2 f01 + 2 n^{12} r^2 f01 + 40 m^6 r^2 f01 \\
& + \frac{140}{3} m^3 n^6 r^4 + 12 m^2 r^2 f01 - \frac{70}{3} m^3 n^8 r^4 - 20 m^7 n^4 r^4 + \frac{35}{3} m^3 n^2 r^4 - \frac{15}{2} n^{16} m^5 r^4 \\
& - \frac{1}{6} m^{21} r^4 - \frac{1}{6} m r^4 - \frac{100}{3} m^{15} n^2 r^4 - \frac{140}{3} m^{13} n^6 r^4 - 42 n^{10} m^{11} r^4 - 20 n^6 m^{15} r^4 \\
& - \frac{15}{2} n^4 m^{17} r^4 - \frac{5}{3} n^2 m^{19} r^4 - 35 n^8 m^{13} r^4 - 20 m^{11} n^4 r^4 - \frac{70}{3} m^3 n^{10} r^4 \\
& - \frac{100}{3} m^{15} n^4 r^4 - \frac{35}{3} m^{17} n^2 r^4 + \frac{70}{3} m^9 n^{10} r^4 - \frac{140}{3} m^{13} n^2 r^4 - \frac{70}{3} m^{11} n^8 r^4 \\
& + \frac{25}{3} m^9 n^8 r^4 - 20 m^{11} n^6 r^4 + \frac{140}{3} m^7 n^{12} r^4 - 20 n^{14} m^7 r^4 + \frac{140}{3} m^7 n^2 r^4 \\
& + \frac{35}{3} m^3 n^{16} r^4 + \frac{25}{3} m^5 n^8 r^4 + 20 m^5 n^{10} r^4 - \frac{100}{3} m^3 n^{14} r^4 - 50 m^5 n^{12} r^4
\end{aligned}$$



$$\begin{aligned}
& -\frac{20}{3} m^7 n^8 r^4 - \frac{20}{3} m^7 n^6 r^4 + \frac{25}{3} m^9 n^4 r^4 + \frac{140}{3} m^3 n^{12} r^4 + \frac{20}{3} m^9 n^6 r^4 - 20 m^7 n^{10} r^4 \\
& -\frac{70}{3} m^{11} n^2 r^4 - 50 m^{13} n^4 r^4 + \frac{100}{3} m^5 n^{14} r^4 + \frac{100}{3} m^5 n^2 r^4, G001 = -\frac{1}{24} r (-5 m^{14} r^2 \\
& + 3 n^{14} r^2 + 24 m^5 f01 + 24 f01 m + 48 m^3 f01 - 3 r^2 - 145 m^8 r^2 - 93 m^{10} r^2
\end{aligned}$$

$$\begin{aligned}
& -23 m^2 r^2 - 75 m^4 r^2 - 33 m^{12} r^2 - 135 m^6 r^2 + 105 n^6 r^2 + 63 n^{10} r^2 - 105 n^8 r^2 \\
& -63 n^4 r^2 + 21 n^2 r^2 - 21 n^{12} r^2 - 42 m^4 n^6 r^2 - 69 m^8 n^2 r^2 - 78 m^4 n^4 r^2 \\
& + 30 m^6 n^4 r^2 + 102 m^2 n^2 r^2 + 171 m^4 n^2 r^2 + 15 m^2 n^8 r^2 + 100 m^2 n^6 r^2 + 84 m^6 n^2 r^2 \\
& -165 m^2 n^4 r^2 - 42 n^{10} m^2 r^2 + 9 n^8 m^4 r^2 - 51 n^4 m^8 r^2 - 90 n^2 m^{10} r^2 + 36 n^6 m^6 r^2
\end{aligned}$$

$$\begin{aligned}
& + 48 m^3 n^2 f01 - 48 n^2 f01 m + 24 n^4 f01 m - 27 m^{12} n^2 r^2 + 13 m^2 n^{12} r^2 - 15 m^6 n^8 r^2 \\
& + 15 m^4 n^{10} r^2 - 55 m^8 n^6 r^2 - 57 m^{10} n^4 r^2)/m, F01 = \frac{1}{12} r^2 + \frac{1}{12} m^8 r^2 + \frac{1}{3} m^2 r^2 \\
& + \frac{1}{2} m^4 r^2 + \frac{1}{3} m^6 r^2 - \frac{1}{3} n^6 r^2 + \frac{1}{12} n^8 r^2 + \frac{1}{2} n^4 r^2 - \frac{1}{3} n^2 r^2 + \frac{1}{2} m^4 n^4 r^2 - \frac{1}{3} m^2 n^2 r^2
\end{aligned}$$

$$+ \frac{1}{3} m^4 n^2 r^2 + \frac{1}{3} m^2 n^6 r^2 + \frac{1}{3} m^6 n^2 r^2 - \frac{1}{3} m^2 n^4 r^2 \}$$

>

## Second Step: (4,5)-normalization -----

### (3,4,5) jet

```

> restart:readlib(mttaylor):
A:=0:B:=0:x:=(I*z-I*cz)/2:y:=(I*z+I*cz)/(2*I):
S:=exp(x+m*y)*sin(n*y):T:=exp(x+m*y)*cos(n*y):
S1:=mtaylor(2*S/m/n,[z,cz],8): T1:=mtaylor(16*T,[z,cz],9):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=S2:
T3:=-2*m*(T2+(4*n^2-4-4*m^2)*S2):
k:=(m^2+1-2*n+n^2)*(m^2+1+2*n+n^2):
ck:=(m^2+1-2*n+n^2)*(m^2+1+2*n+n^2):
M:=k*ck:

```

**Write the results of the previous step.**

```

> f01:= a:
F01:=1/12*m^8*r^2+1/3*m^6*r^2+1/2*m^4*r^2+1/3*m^2*r^2+1/2*n^4*r^2-
1/3*n^2*r^2+1/12*n^8*r^2-
1/3*n^6*r^2+1/12*r^2+1/3*m^6*n^2*r^2+1/3*m^4*n^2*r^2-
1/3*m^2*n^4*r^2+1/2*m^4*n^4*r^2+1/3*m^2*n^6*r^2-1/3*m^2*n^2*r^2:
h4:= 0: H101:= 0: h101:= 0: g001:= 0: g3:= 0: H4:= 0: G3:= 0:

```

```

G11:= 2*r*f01+4*m^2*r*f01-
4*n^2*r*f01+2*m^4*r*f01+4*m^2*n^2*r*f01+2*n^4*r*f01:
h21:=35*n^12*r^4+15/2*n^4*m^16*r^4+15/2*n^16*m^4*r^4-
20*m^4*n^10*r^4+50*m^4*n^12*r^4-25/3*m^4*n^8*r^4+20/3*m^6*n^8*r^4-
20/3*m^8*n^6*r^4+20*m^10*n^6*r^4+20*m^6*n^10*r^4-
42*n^10*r^4+35*n^8*m^12*r^4+70/3*m^10*n^2*r^4+100/3*m^14*n^2*r^4-
100/3*m^4*n^14*r^4+35*n^8*r^4-20*n^6*r^4-
140/3*m^6*n^12*r^4+140/3*m^12*n^6*r^4+15/2*n^4*r^4+5/3*m^18*r^4-
70/3*m^8*n^10*r^4+70/3*m^10*n^8*r^4+15/2*m^16*r^4+140/3*m^12*n^2*r^4
4+100/3*m^2*n^14*r^4+35/3*m^16*n^2*r^4+20*m^14*r^4-
140/3*m^2*n^12*r^4+5/3*n^18*m^2*r^4+5/3*n^2*m^18*r^4+20*n^6*m^14*r^4
4+35*m^12*r^4+42*m^10*r^4+35*m^8*r^4+20*m^6*r^4+15/2*m^4*r^4+5/3*m^2
r^4+1/6*n^20*r^4+1/6*m^20*r^4+20*n^14*m^6*r^4+35*n^12*m^8*r^4+42*
n^10*m^10*r^4+1/6*r^4+50*m^4*n^4*r^4-25/3*m^8*n^8*r^4-
5/3*n^18*r^4+15/2*n^16*r^4+70/3*m^2*n^8*r^4-70/3*m^8*n^2*r^4-
20*m^4*n^6*r^4+50*m^12*n^4*r^4-5/3*n^2*r^4-
20*n^14*r^4+20/3*m^6*n^6*r^4+70/3*m^2*n^10*r^4-35/3*m^2*n^16*r^4-
25/3*m^8*n^4*r^4+20*m^10*n^4*r^4+100/3*m^14*n^4*r^4+20*m^6*n^4*r^4-
35/3*m^2*n^2*r^4+100/3*m^2*n^4*r^4-100/3*m^4*n^2*r^4-
140/3*m^6*n^2*r^4-140/3*m^2*n^6*r^4:
H02:=35*n^12*r^4+15/2*n^4*m^16*r^4+15/2*n^16*m^4*r^4-
20*m^4*n^10*r^4+50*m^4*n^12*r^4-25/3*m^4*n^8*r^4+20/3*m^6*n^8*r^4-
20/3*m^8*n^6*r^4+20*m^10*n^6*r^4+20*m^6*n^10*r^4-
42*n^10*r^4+35*n^8*m^12*r^4+70/3*m^10*n^2*r^4+100/3*m^14*n^2*r^4-
100/3*m^4*n^14*r^4+35*n^8*r^4-20*n^6*r^4-
140/3*m^6*n^12*r^4+140/3*m^12*n^6*r^4+15/2*n^4*r^4+5/3*m^18*r^4-
70/3*m^8*n^10*r^4+70/3*m^10*n^8*r^4+15/2*m^16*r^4+140/3*m^12*n^2*r^4
4+100/3*m^2*n^14*r^4+35/3*m^16*n^2*r^4+20*m^14*r^4-
140/3*m^2*n^12*r^4+5/3*n^18*m^2*r^4+5/3*n^2*m^18*r^4+20*n^6*m^14*r^4
4+35*m^12*r^4+42*m^10*r^4+35*m^8*r^4+20*m^6*r^4+15/2*m^4*r^4+5/3*m^2
r^4+1/6*n^20*r^4+1/6*m^20*r^4+20*n^14*m^6*r^4+35*n^12*m^8*r^4+42*
n^10*m^10*r^4+1/6*r^4+50*m^4*n^4*r^4-25/3*m^8*n^8*r^4-
5/3*n^18*r^4+15/2*n^16*r^4+70/3*m^2*n^8*r^4-70/3*m^8*n^2*r^4-
20*m^4*n^6*r^4+50*m^12*n^4*r^4-5/3*n^2*r^4-
20*n^14*r^4+20/3*m^6*n^6*r^4+70/3*m^2*n^10*r^4-35/3*m^2*n^16*r^4-
25/3*m^8*n^4*r^4+20*m^10*n^4*r^4+100/3*m^14*n^4*r^4+20*m^6*n^4*r^4-
35/3*m^2*n^2*r^4+100/3*m^2*n^4*r^4-100/3*m^4*n^2*r^4-
140/3*m^6*n^2*r^4-140/3*m^2*n^6*r^4:
H21:=12*n^2*m^10*r^2*f01+30*n^8*m^4*r^2*f01+12*n^10*m^2*r^2*f01+30*
m^4*r^2*f01+40*n^6*m^6*r^2*f01+2*n^12*r^2*f01+30*n^4*m^8*r^2*f01+2*
m^12*r^2*f01+40*m^6*r^2*f01+2*r^2*f01+12*m^2*r^2*f01+12*m^10*r^2*f0
1+24*m^2*n^4*r^2*f01-
12*n^10*r^2*f01+30*n^8*r^2*f01+36*m^8*n^2*r^2*f01+24*m^6*n^4*r^2*f0
1+24*m^6*n^2*r^2*f01-40*n^6*r^2*f01-24*m^4*n^2*r^2*f01-
36*m^2*n^2*r^2*f01+30*m^8*r^2*f01-
12*m^4*n^4*r^2*f01+24*m^2*n^6*r^2*f01-24*m^4*n^6*r^2*f01-
36*m^2*n^8*r^2*f01-12*n^2*r^2*f01+30*n^4*r^2*f01: f2:=
1/12*m^8*r^2-1/3*m^6*n^2*r^2-1/3*m^6*r^2-1/2*m^4*n^4*r^2-
1/3*m^4*n^2*r^2-1/2*m^4*r^2+1/3*m^2*n^4*r^2+1/3*m^2*n^2*r^2-

```

$1/3*m^2*r^2-1/3*m^2*n^6*r^2-1/12*n^8*r^2+1/3*n^2*r^2-$   
 $1/2*n^4*r^2+1/3*n^6*r^2-1/12*r^2:$   
 $F2:=-1/6*m^9*r^2-2/3*m^7*n^2*r^2-2/3*m^7*r^2-m^5*n^4*r^2-$   
 $2/3*m^5*n^2*r^2-m^5*r^2+2/3*m^3*n^4*r^2+2/3*m^3*n^2*r^2-$   
 $2/3*m^3*r^2-2/3*m^3*n^6*r^2-1/6*m*n^8*r^2+2/3*m*n^2*r^2-$   
 $m*n^4*r^2+2/3*m*n^6*r^2-1/6*m*r^2+f01:$   
 $g11:=1/6*r^3+m^10*r^3+1/6*m^12*r^3+5/2*m^8*r^3+10/3*m^6*r^3+5/2*m^4$   
 $*r^3+m^2*r^3+m^10*n^2*r^3+5/2*n^4*r^3-n^2*r^3+5/2*n^8*r^3-$   
 $10/3*n^6*r^3+2*m^6*n^2*r^3-2*m^4*n^2*r^3+2*m^2*n^4*r^3-$   
 $m^4*n^4*r^3+2*m^2*n^6*r^3-$   
 $3*m^2*n^2*r^3+5/2*m^4*n^8*r^3+1/6*n^12*r^3-n^10*r^3+2*m^6*n^4*r^3-$   
 $2*m^4*n^6*r^3-$   
 $3*m^2*n^8*r^3+m^2*n^10*r^3+10/3*m^6*n^6*r^3+3*m^8*n^2*r^3+5/2*m^8*n$   
 $^4*r^3: G001:=1/24*r*(-3*n^14*r^2+5*m^14*r^2-$   
 $63*n^10*r^2+93*m^10*r^2+33*m^12*r^2+145*m^8*r^2+135*m^6*r^2+75*m^4*$   
 $r^2+23*m^2*r^2+63*n^4*r^2-21*n^2*r^2+105*n^8*r^2-$   
 $105*n^6*r^2+21*n^12*r^2+3*r^2+90*m^10*n^2*r^2-84*m^6*n^2*r^2-$   
 $171*m^4*n^2*r^2+165*m^2*n^4*r^2+78*m^4*n^4*r^2-100*m^2*n^6*r^2-$   
 $102*m^2*n^2*r^2-9*m^4*n^8*r^2-30*m^6*n^4*r^2+42*m^4*n^6*r^2-$   
 $15*m^2*n^8*r^2+42*m^2*n^10*r^2-$   
 $36*m^6*n^6*r^2+69*m^8*n^2*r^2+51*m^8*n^4*r^2-$   
 $13*m^2*n^12*r^2+27*m^12*n^2*r^2-$   
 $15*m^4*n^10*r^2+15*m^6*n^8*r^2+55*m^8*n^6*r^2+57*m^10*n^4*r^2-$   
 $48*m^3*n^2*f01+48*n^2*f01*m-24*n^4*f01*m-24*f01*m-48*m^3*f01-$   
 $24*m^5*f01)/m:$   
 $h02:=-35*m*n^8*r^4+5/3*m*n^2*r^4-$   
 $15/2*m*n^4*r^4+25/3*m^9*n^4*r^4+20*m*n^6*r^4-$   
 $100/3*m^3*n^4*r^4+20/3*m^9*n^6*r^4-50*m^13*n^4*r^4-$   
 $100/3*m^15*n^4*r^4+20*m^5*n^10*r^4-15/2*m^17*r^4-5/3*m^19*r^4-$   
 $5/3*m^3*r^4-35*m^13*r^4-20*m^15*r^4-20*m^7*r^4-15/2*m^5*r^4-$   
 $35*m^9*r^4-70/3*m^11*n^8*r^4+70/3*m^9*n^10*r^4-140/3*m^13*n^6*r^4-$   
 $20*m^11*n^6*r^4+25/3*m^9*n^8*r^4-20/3*m^7*n^8*r^4-$   
 $100/3*m^3*n^14*r^4+35/3*m^3*n^16*r^4-100/3*m^15*n^2*r^4-$   
 $35/3*m^17*n^2*r^4+100/3*m^5*n^14*r^4+12*n^2*m^10*r^2*f01+30*n^8*m^4$   
 $*r^2*f01+12*n^10*m^2*r^2*f01+30*m^4*r^2*f01-$   
 $1/6*m*r^4+40*n^6*m^6*r^2*f01+2*n^12*r^2*f01+30*n^4*m^8*r^2*f01-$   
 $5/3*n^18*m^3*r^4+140/3*m^7*n^2*r^4-$   
 $50*m^5*n^4*r^4+2*m^12*r^2*f01+140/3*m^3*n^12*r^4-$   
 $140/3*m^13*n^2*r^4-20*m^7*n^4*r^4+20*m^5*n^6*r^4+70/3*m^9*n^2*r^4-$   
 $1/6*m*n^20*r^4+42*m*n^10*r^4+40*m^6*r^2*f01-20/3*m^7*n^6*r^4-$   
 $20*m^7*n^10*r^4+140/3*m^7*n^12*r^4-15/2*n^16*m^5*r^4-$   
 $5/3*n^2*m^19*r^4-$   
 $20*n^6*m^15*r^4+35/3*m^3*n^2*r^4+140/3*m^3*n^6*r^4-$   
 $42*n^10*m^11*r^4-35*n^12*m^9*r^4-15/2*n^4*m^17*r^4+2*r^2*f01-$   
 $20*m^11*n^4*r^4-35*n^8*m^13*r^4-70/3*m^3*n^8*r^4+100/3*m^5*n^2*r^4-$   
 $70/3*m^11*n^2*r^4+25/3*m^5*n^8*r^4-70/3*m^3*n^10*r^4-$   
 $50*m^5*n^12*r^4+12*m^2*r^2*f01-42*m^11*r^4-$   
 $20*n^14*m^7*r^4+20*m*n^14*r^4+5/3*m*n^18*r^4-15/2*m*n^16*r^4-$   
 $35*m*n^12*r^4+12*m^10*r^2*f01+24*m^2*n^4*r^2*f01-$

```

12*n^10*r^2*f01+30*n^8*r^2*f01+36*m^8*n^2*r^2*f01+24*m^6*n^4*r^2*f0
1+24*m^6*n^2*r^2*f01-40*n^6*r^2*f01-24*m^4*n^2*r^2*f01-
36*m^2*n^2*r^2*f01+30*m^8*r^2*f01-
12*m^4*n^4*r^2*f01+24*m^2*n^6*r^2*f01-24*m^4*n^6*r^2*f01-
36*m^2*n^8*r^2*f01-12*n^2*r^2*f01+30*n^4*r^2*f01-1/6*m^21*r^4:
> w2:=u2+I*(z*cz+B*z^2*cz^2): cw2:=u2-I*(z*cz+B*z^2*cz^2):
w3:=u3+I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+A*z*cz+(C1+I*C2)*z^3*cz^2+(C1-
I*C2)*cz^3*z^2):

```

### Write a mapping

```

> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:

g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:

cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:

h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:

ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:

```

**Write the relations for the mapping.**

```
> e2:=subs ([z=f, cz=cf], 2*S3)+I*(g-cg) :  
e3:=subs ([z=f, cz=cf], 2*T3)+I*(h-ch) :
```

**Write the coefficients of the (4,5)-jet.**

```
> P[4,0,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z, z))):  
P[3,1,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, z, cz))):  
P[2,2,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, cz, cz))):  
P[2,0,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, z, u2))):  
P[1,1,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, cz, u2))):  
P[1,0,0,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e2, z, u3))):
```

```
P1[4,0,0,0]:=subs (I=0, P[4,0,0,0]) : P2[4,0,0,0]:=expand((P[4,0,0,0]-  
P1[4,0,0,0])*I) :  
P1[3,1,0,0]:=subs (I=0, P[3,1,0,0]) : P2[3,1,0,0]:=expand((P[3,1,0,0]-  
P1[3,1,0,0])*I) :  
P1[2,0,1,0]:=subs (I=0, P[2,0,1,0]) : P2[2,0,1,0]:=expand((P[2,0,1,0]-  
P1[2,0,1,0])*I) :  
P1[1,0,0,1]:=subs (I=0, P[1,0,0,1]) : P2[1,0,0,1]:=expand((P[1,0,0,1]-  
P1[1,0,0,1])*I) :
```

```
Q[5,0,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, z, z))):  
Q[4,1,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, z, cz))):  
Q[3,2,0,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, cz, cz))):  
:  
Q[3,0,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, z, u2))):  
Q[2,1,1,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, cz, u2))):  
Q[1,0,2,0]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, u2, u2))):  
Q[2,0,0,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, z, u3))):  
Q[1,1,0,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, z, cz, u3))):  
Q[0,0,1,1]:=expand(subs ([z=0, cz=0, u2=0, u3=0], diff(e3, u2, u3))):
```

```
Q1[5,0,0,0]:=subs (I=0, Q[5,0,0,0]) : Q2[5,0,0,0]:=expand((Q[5,0,0,0]-  
Q1[5,0,0,0])*I) :  
Q1[4,1,0,0]:=subs (I=0, Q[4,1,0,0]) : Q2[4,1,0,0]:=expand((Q[4,1,0,0]-  
Q1[4,1,0,0])*I) :  
Q1[3,2,0,0]:=subs (I=0, Q[3,2,0,0]) : Q2[3,2,0,0]:=expand((Q[3,2,0,0]-  
Q1[3,2,0,0])*I) :  
Q1[3,0,1,0]:=subs (I=0, Q[3,0,1,0]) : Q2[3,0,1,0]:=expand((Q[3,0,1,0]-  
Q1[3,0,1,0])*I) :  
Q1[2,1,1,0]:=subs (I=0, Q[2,1,1,0]) : Q2[2,1,1,0]:=expand((Q[2,1,1,0]-  
Q1[2,1,1,0])*I) :  
Q1[1,0,2,0]:=subs (I=0, Q[1,0,2,0]) : Q2[1,0,2,0]:=expand((Q[1,0,2,0]-  
Q1[1,0,2,0])*I) :  
Q1[2,0,0,1]:=subs (I=0, Q[2,0,0,1]) : Q2[2,0,0,1]:=expand((Q[2,0,0,1]-  
Q1[2,0,0,1])*I) :
```

**Write the equations and the variables.**

```
> EQ:={  
P1[4,0,0,0]=0, P2[4,0,0,0]=0,
```

$P1[3, 1, 0, 0]=0, P2[3, 1, 0, 0]=0,$   
 $P[2, 2, 0, 0]=0,$   
 $P1[2, 0, 1, 0]=0, P2[2, 0, 1, 0]=0,$   
 $P[1, 1, 1, 0]=0,$   
 $P1[1, 0, 0, 1]=0, P2[1, 0, 0, 1]=0,$

$Q1[5, 0, 0, 0]=0, Q2[5, 0, 0, 0]=0,$   
 $Q1[4, 1, 0, 0]=0, Q2[4, 1, 0, 0]=0,$   
 $Q1[3, 2, 0, 0]=0, Q2[3, 2, 0, 0]=0,$   
 $Q1[3, 0, 1, 0]=0, Q2[3, 0, 1, 0]=0,$   
 $Q1[2, 1, 1, 0]=0, Q2[2, 1, 1, 0]=0,$   
 $Q1[1, 0, 2, 0]=0, Q2[1, 0, 2, 0]=0,$   
 $Q1[2, 0, 0, 1]=0, Q2[2, 0, 0, 1]=0,$   
 $Q[1, 1, 0, 1]=0,$   
 $Q[0, 0, 1, 1]=0$   
 }:

**Number of equations - 26.**

> sol:=solve(EQ) :

> sol[2];

$$\begin{aligned}
 \{g101 = & -\frac{1}{72} m r^4 - \frac{1}{6} r^2 a - \frac{1}{2} m^5 n^4 r^4 + \frac{5}{9} m^3 n^8 r^4 + \frac{5}{9} m^3 n^6 r^4 + \frac{5}{9} m^3 n^{12} r^4 - \frac{2}{3} m^6 r^2 a \\
 & - \frac{2}{3} m^2 r^2 a - \frac{1}{6} n^8 r^2 a + \frac{2}{3} n^6 r^2 a - \frac{1}{6} m^8 r^2 a + \frac{2}{3} n^2 r^2 a - n^4 r^2 a - m^4 r^2 a - m^3 n^4 r^4 \\
 & - \frac{7}{18} m^{13} n^4 r^4 - \frac{35}{36} m^9 n^8 r^4 + \frac{5}{9} m^7 n^2 r^4 - \frac{1}{9} m^{15} n^2 r^4 - \frac{1}{9} m^3 n^{14} r^4 + \frac{5}{9} m^7 n^8 r^4 \\
 & - \frac{5}{9} m^9 n^6 r^4 - \frac{35}{36} m n^8 r^4 - m^{11} n^4 r^4 - \frac{7}{18} m n^4 r^4 - \frac{1}{2} m^9 n^4 r^4 + \frac{7}{9} m n^6 r^4 + m^5 n^{10} r^4 \\
 & - \frac{7}{9} m^{11} n^6 r^4 - \frac{5}{9} m^{13} n^2 r^4 - \frac{7}{9} m^7 n^{10} r^4 + \frac{5}{9} m^3 n^2 r^4 + m^5 n^2 r^4 - m^{11} n^2 r^4 \\
 & - \frac{1}{2} m^5 n^8 r^4 - m^3 n^{10} r^4 + \frac{1}{9} m n^2 r^4 + \frac{2}{3} m^2 n^4 r^2 a - \frac{7}{18} m^5 r^4 - \frac{35}{36} m^9 r^4 - \frac{7}{9} m^{11} r^4 \\
 & - \frac{2}{3} m^4 n^2 r^2 a - \frac{7}{9} m^7 r^4 - \frac{2}{3} m^6 n^2 r^2 a - \frac{1}{9} m^3 r^4 - \frac{7}{18} m^{13} r^4 - \frac{1}{9} m^{15} r^4 - \frac{7}{18} m^5 n^{12} r^4 \\
 & - \frac{1}{72} m^{17} r^4 + \frac{1}{9} m n^{14} r^4 - \frac{1}{72} m n^{16} r^4 - \frac{7}{18} m n^{12} r^4 + \frac{7}{9} m n^{10} r^4 + \frac{2}{9} m^7 n^4 r^4
 \end{aligned}$$

$$\begin{aligned}
& + \frac{2}{3} m^2 n^2 r^2 a - m^4 n^4 r^2 a - \frac{2}{3} m^2 n^6 r^2 a + \frac{2}{9} m^7 n^6 r^4 - \frac{5}{9} m^9 n^2 r^4 - \frac{2}{9} m^5 n^6 r^4, f001 \\
& = - \frac{1}{288} (228 m^8 n^6 r^4 + 540 m^{10} n^4 r^4 + 96 m^{14} n^2 r^4 - 384 m^2 n^{14} r^4 + 252 m^4 n^6 r^4 \\
& + 244 m^{12} n^2 r^4 + 510 m^8 n^2 r^4 + 308 m^{12} n^6 r^4 - 648 m^4 n^4 r^4 + 100 m^{14} n^4 r^4
\end{aligned}$$

$$\begin{aligned}
& + 512 m^6 n^2 r^4 + 574 m^{10} n^8 r^4 + 400 m^{10} n^2 r^4 + 180 m^8 n^8 r^4 - 28 m^6 n^8 r^4 \\
& + 376 m^{12} n^4 r^4 + 32 m^4 n^8 r^4 - 210 m^2 n^8 r^4 - 544 m^6 n^{10} r^4 + 73 m^2 n^{16} r^4 \\
& + 576 m^{10} n^6 r^4 + 540 m^4 n^{10} r^4 + 532 m^6 n^{12} r^4 + 756 m^2 n^{12} r^4 - 560 n^{12} r^4 \\
& + 742 n^{10} r^4 - 644 n^8 r^4 + 364 n^6 r^4 - 128 n^4 r^4 + m^{18} r^4 + 6 m^{16} r^4 + 12 m^{14} r^4
\end{aligned}$$

$$\begin{aligned}
& - 42 m^{10} r^4 - 84 m^8 r^4 - 84 m^6 r^4 - 48 m^4 r^4 - 15 m^2 r^4 + 9 n^{18} r^4 - 74 n^{16} r^4 \\
& + 25 n^2 r^4 + 268 n^{14} r^4 + 672 m^2 n^6 r^4 - 476 m^2 n^4 r^4 + 144 m^2 n^2 r^4 - 96 m^6 n^6 r^4 \\
& + 260 m^4 n^{14} r^4 + 356 m^4 n^2 r^4 - 292 m^6 n^4 r^4 + 272 m^8 n^4 r^4 - 560 m^2 n^{10} r^4 \\
& + 17 m^{16} n^2 r^4 + 686 m^8 n^{10} r^4 - 744 m^4 n^{12} r^4 - 2 r^4 + 144 a^2) / (
\end{aligned}$$

$$r(m^2 + 1 - 2n + n^2)(m^2 + 1 + 2n + n^2), g4 = 0, G4 = 0, h011 = 0, H5 = 0, h5 = 0,$$

$$h31 = \frac{56}{3} r^3 m^{12} a n^4 + \frac{112}{3} r^3 m^{10} a n^6 + \frac{32}{3} r^3 n^6 m^4 a + \frac{140}{3} r^3 m^8 a n^8$$

$$- 48 r^3 a m^4 n^{10} + 24 r^3 n^8 m^4 a - \frac{32}{3} r^3 m^6 a n^6 - \frac{80}{3} r^3 a m^6 n^8 + \frac{80}{3} r^3 m^{12} a n^2$$

$$- \frac{32}{3} r^3 m^6 a n^4 - \frac{80}{3} r^3 a m^6 n^2 + \frac{80}{3} r^3 m^8 a n^2 + 24 r^3 m^8 a n^4 + 48 r^3 m^{10} a n^2$$

$$+ 24 r^3 a m^4 n^4 + \frac{2}{3} r^3 m^{16} a + \frac{16}{3} r^3 m^{14} a + \frac{140}{3} r^3 a m^8 + \frac{2}{3} n^{16} r^3 a + \frac{112}{3} m^{10} r^3 a$$

$$+ \frac{56}{3} r^3 m^{12} a - \frac{112}{3} n^{10} r^3 a - \frac{16}{3} n^2 r^3 a + \frac{56}{3} n^4 r^3 a + \frac{56}{3} r^3 a m^4 + \frac{140}{3} n^8 r^3 a$$

$$+ \frac{56}{3} n^{12} r^3 a - \frac{16}{3} n^{14} r^3 a - \frac{112}{3} n^6 r^3 a + \frac{112}{3} r^3 m^6 a + \frac{16}{3} m^2 r^3 a + \frac{2}{3} r^3 a$$

$$\begin{aligned}
& + \frac{16}{3} m^2 r^3 n^{14} a - \frac{80}{3} m^2 r^3 a n^6 - \frac{80}{3} m^2 r^3 n^{12} a - 48 r^3 n^2 m^4 a - \frac{80}{3} m^2 n^8 r^3 a \\
& + 48 m^2 r^3 n^{10} a + 48 m^2 r^3 n^4 a - \frac{80}{3} m^2 r^3 a n^2 + \frac{80}{3} r^3 m^8 a n^6 + 48 r^3 m^{10} a n^4 \\
& + \frac{16}{3} r^3 m^{14} a n^2 + \frac{112}{3} r^3 n^{10} a m^6 + \frac{56}{3} r^3 n^{12} a m^4, H12 = \frac{28}{3} r^3 m^{12} a n^4 \\
& + \frac{56}{3} r^3 m^{10} a n^6 + \frac{16}{3} r^3 n^6 m^4 a + \frac{70}{3} r^3 m^8 a n^8 - 24 r^3 a m^4 n^{10} + 12 r^3 n^8 m^4 a \\
& - \frac{16}{3} r^3 m^6 a n^6 - \frac{40}{3} r^3 a m^6 n^8 + \frac{40}{3} r^3 m^{12} a n^2 - \frac{16}{3} r^3 m^6 a n^4 - \frac{40}{3} r^3 a m^6 n^2 \\
& + \frac{40}{3} r^3 m^8 a n^2 + 12 r^3 m^8 a n^4 + 24 r^3 m^{10} a n^2 + 12 r^3 a m^4 n^4 + \frac{1}{3} r^3 m^{16} a \\
& + \frac{8}{3} r^3 m^{14} a + \frac{70}{3} r^3 a m^8 + \frac{1}{3} n^{16} r^3 a + \frac{56}{3} m^{10} r^3 a + \frac{28}{3} r^3 m^{12} a - \frac{56}{3} n^{10} r^3 a \\
& - \frac{8}{3} n^2 r^3 a + \frac{28}{3} n^4 r^3 a + \frac{28}{3} r^3 a m^4 + \frac{70}{3} n^8 r^3 a + \frac{28}{3} n^{12} r^3 a - \frac{8}{3} n^{14} r^3 a - \frac{56}{3} n^6 r^3 a \\
& + \frac{56}{3} r^3 m^6 a + \frac{8}{3} m^2 r^3 a + \frac{1}{3} r^3 a + \frac{8}{3} m^2 r^3 n^{14} a - \frac{40}{3} m^2 r^3 a n^6 - \frac{40}{3} m^2 r^3 n^{12} a \\
& - 24 r^3 n^2 m^4 a - \frac{40}{3} m^2 n^8 r^3 a + 24 m^2 r^3 n^{10} a + 24 m^2 r^3 n^4 a - \frac{40}{3} m^2 r^3 a n^2 \\
& + \frac{40}{3} r^3 m^8 a n^6 + 24 r^3 m^{10} a n^4 + \frac{8}{3} r^3 m^{14} a n^2 + \frac{56}{3} r^3 n^{10} a m^6 + \frac{28}{3} r^3 n^{12} a m^4, h12 \\
& = \frac{35}{2} r^5 n^{12} m^8 + 50 r^5 n^4 m^{14} + 45 r^5 n^{14} m^2 + \frac{55}{6} r^5 m^{18} n^6 + \frac{1}{24} r^5 m^{24} + \frac{55}{6} r^5 m^{18} \\
& + \frac{1}{24} r^5 + \frac{165}{8} r^5 m^{16} + 8 n^2 m^6 r a^2 + 8 n^6 m^2 r a^2 + 12 n^4 m^4 r a^2 + 2 n^8 r a^2 \\
& - 5 r^5 n^6 m^{10} - 7 r^5 n^4 m^{10} + \frac{77}{2} r^5 m^{12} n^{12} + 5 r^5 n^6 m^8 - \frac{55}{6} n^{18} r^5 + \frac{1}{2} r^5 n^{22} m^2
\end{aligned}$$



$$\begin{aligned}
& -21 r^5 n^{12} m^2 + \frac{35}{2} r^5 n^{18} m^2 + 2 m^8 r a^2 + 45 r^5 n^2 m^{14} + 7 r^5 n^{10} m^4 - 5 r^5 n^8 m^{10} \\
& - \frac{75}{2} r^5 n^{16} m^2 + \frac{1}{24} n^{24} r^5 - \frac{55}{6} n^6 r^5 - \frac{1}{2} n^2 r^5 + \frac{11}{4} n^4 r^5 + \frac{11}{4} n^{20} r^5 - \frac{1}{2} n^{22} r^5 \\
& + 8 m^6 r a^2 + \frac{165}{8} n^{16} r^5 - 33 n^{14} r^5 + \frac{77}{2} n^{12} r^5 - 33 n^{10} r^5 + \frac{165}{8} n^8 r^5 + \frac{1}{2} r^5 m^{22} \\
& + 33 r^5 m^{14} + \frac{77}{2} r^5 m^{12} + \frac{11}{4} r^5 m^4 + \frac{1}{2} r^5 m^2 + 33 r^5 m^{10} + \frac{165}{8} r^5 m^8 + \frac{55}{6} r^5 m^6 \\
& + \frac{11}{4} r^5 m^{20} + 8 m^2 r a^2 + 2 r a^2 + 12 m^4 r a^2 + \frac{35}{2} r^5 n^{12} m^4 + \frac{75}{2} r^5 n^2 m^{16} \\
& - 50 r^5 n^{14} m^4 - 5 r^5 n^{10} m^6 - 5 r^5 n^8 m^6 - \frac{50}{3} r^5 n^{12} m^6 + \frac{15}{4} r^5 n^8 m^8 + 5 r^5 n^{10} m^8 \\
& + \frac{35}{2} r^5 m^{18} n^2 + \frac{175}{4} r^5 n^4 m^{16} + \frac{175}{4} r^5 n^{16} m^4 + 50 r^5 n^6 m^{14} + \frac{35}{2} r^5 m^{18} n^4 \\
& + \frac{165}{8} r^5 n^8 m^{16} + \frac{75}{2} r^5 n^6 m^{16} + 50 r^5 n^{14} m^6 - \frac{35}{2} r^5 n^{18} m^4 + 45 r^5 n^8 m^{14} \\
& + \frac{1}{2} r^5 m^{22} n^2 + \frac{11}{4} r^5 n^4 m^{20} - 45 r^5 n^{14} m^8 - \frac{75}{2} r^5 n^{16} m^6 + \frac{9}{2} r^5 n^2 m^{20} - 21 r^5 n^{12} m^{10} \\
& - 7 r^5 n^{10} m^{10} + \frac{55}{6} r^5 n^{18} m^6 + \frac{11}{4} r^5 n^{20} m^4 + \frac{165}{8} r^5 n^{16} m^8 + 21 r^5 m^{12} n^2 \\
& + \frac{35}{2} r^5 n^8 m^{12} + \frac{50}{3} r^5 m^{12} n^6 + \frac{35}{2} r^5 m^{12} n^4 + \frac{175}{4} r^5 n^4 m^4 + \frac{35}{2} r^5 n^8 m^4 - \frac{50}{3} r^5 n^6 m^6 \\
& - 50 r^5 n^6 m^4 + 50 r^5 n^4 m^6 + \frac{35}{2} r^5 n^4 m^8 - \frac{9}{2} r^5 n^2 m^2 - 21 r^5 n^2 m^{10} - 45 r^5 n^2 m^8 \\
& - \frac{75}{2} r^5 n^2 m^6 - \frac{35}{2} r^5 n^2 m^4 - \frac{75}{2} r^5 n^6 m^2 + \frac{35}{2} r^5 n^4 m^2 - 21 r^5 n^{10} m^2 + 45 r^5 n^8 m^2 \\
& + 33 r^5 n^{14} m^{10} - \frac{9}{2} r^5 n^{20} m^2 + 21 r^5 m^{12} n^{10} + 33 r^5 n^{10} m^{14} - 8 m^2 n^2 r a^2 - 8 n^6 r a^2
\end{aligned}$$

$$\begin{aligned}
& -8n^2ra^2 + 12n^4ra^2 - 8m^2n^4ra^2 + 8m^4n^2ra^2, H31 = -\frac{35}{3}r^5n^{12}m^8 \\
& -\frac{100}{3}r^5n^4m^{14} - 30r^5n^{14}m^2 - \frac{55}{9}r^5m^{18}n^6 - \frac{1}{36}r^5m^{24} - \frac{55}{9}r^5m^{18} - \frac{1}{36}r^5 \\
& -\frac{55}{4}r^5m^{16} + 16n^2m^6ra^2 + 16n^6m^2ra^2 + 24n^4m^4ra^2 + 4n^8ra^2 + \frac{10}{3}r^5n^6m^{10} \\
& + \frac{14}{3}r^5n^4m^{10} - \frac{77}{3}r^5m^{12}n^{12} - \frac{10}{3}r^5n^6m^8 + \frac{55}{9}n^{18}r^5 - \frac{1}{3}r^5n^{22}m^2 + 14r^5n^{12}m^2 \\
& -\frac{35}{3}r^5n^{18}m^2 + 4m^8ra^2 - 30r^5n^2m^{14} - \frac{14}{3}r^5n^{10}m^4 + \frac{10}{3}r^5n^8m^{10} + 25r^5n^{16}m^2 \\
& -\frac{1}{36}n^{24}r^5 + \frac{55}{9}n^6r^5 + \frac{1}{3}n^2r^5 - \frac{11}{6}n^4r^5 - \frac{11}{6}n^{20}r^5 + \frac{1}{3}n^{22}r^5 + 16m^6ra^2 - \frac{55}{4}n^{16}r^5 \\
& + 22n^{14}r^5 - \frac{77}{3}n^{12}r^5 + 22n^{10}r^5 - \frac{55}{4}n^8r^5 - \frac{1}{3}r^5m^{22} - 22r^5m^{14} - \frac{77}{3}r^5m^{12} \\
& -\frac{11}{6}r^5m^4 - \frac{1}{3}r^5m^2 - 22r^5m^{10} - \frac{55}{4}r^5m^8 - \frac{55}{9}r^5m^6 - \frac{11}{6}r^5m^{20} + 16m^2ra^2 \\
& + 4ra^2 + 24m^4ra^2 - \frac{35}{3}r^5n^{12}m^4 - 25r^5n^2m^{16} + \frac{100}{3}r^5n^{14}m^4 + \frac{10}{3}r^5n^{10}m^6 \\
& + \frac{10}{3}r^5n^8m^6 + \frac{100}{9}r^5n^{12}m^6 - \frac{5}{2}r^5n^8m^8 - \frac{10}{3}r^5n^{10}m^8 - \frac{35}{3}r^5m^{18}n^2 \\
& -\frac{175}{6}r^5n^4m^{16} - \frac{175}{6}r^5n^{16}m^4 - \frac{100}{3}r^5n^6m^{14} - \frac{35}{3}r^5m^{18}n^4 - \frac{55}{4}r^5n^8m^{16} \\
& -25r^5n^6m^{16} - \frac{100}{3}r^5n^{14}m^6 + \frac{35}{3}r^5n^{18}m^4 - 30r^5n^8m^{14} - \frac{1}{3}r^5m^{22}n^2 \\
& -\frac{11}{6}r^5n^4m^{20} + 30r^5n^{14}m^8 + 25r^5n^{16}m^6 - 3r^5n^2m^{20} + 14r^5n^{12}m^{10} \\
& + \frac{14}{3}r^5n^{10}m^{10} - \frac{55}{9}r^5n^{18}m^6 - \frac{11}{6}r^5n^{20}m^4 - \frac{55}{4}r^5n^{16}m^8 - 14r^5m^{12}n^2
\end{aligned}$$

$$\begin{aligned}
& -\frac{35}{3}r^5n^8m^{12} - \frac{100}{9}r^5m^{12}n^6 - \frac{35}{3}r^5m^{12}n^4 - \frac{175}{6}r^5n^4m^4 - \frac{35}{3}r^5n^8m^4 \\
& + \frac{100}{9}r^5n^6m^6 + \frac{100}{3}r^5n^6m^4 - \frac{100}{3}r^5n^4m^6 - \frac{35}{3}r^5n^4m^8 + 3r^5n^2m^2 + 14r^5n^2m^{10}
\end{aligned}$$

$$\begin{aligned}
& + 30r^5n^2m^8 + 25r^5n^2m^6 + \frac{35}{3}r^5n^2m^4 + 25r^5n^6m^2 - \frac{35}{3}r^5n^4m^2 + 14r^5n^{10}m^2 \\
& - 30r^5n^8m^2 - 22r^5n^{14}m^{10} + 3r^5n^{20}m^2 - 14r^5m^{12}n^{10} - 22r^5n^{10}m^{14} \\
& - 16m^2n^2ra^2 - 16n^6ra^2 - 16n^2ra^2 + 24n^4ra^2 - 16m^2n^4ra^2 + 16m^4n^2ra^2,
\end{aligned}$$

$$\begin{aligned}
g02 = & -\frac{1}{72}(-4mr^4 + 27m^{17}n^2r^4 - 18r^2a - 144m^5n^4r^4 + 250m^3n^8r^4 \\
& + 208m^3n^6r^4 + 460m^3n^{12}r^4 - 540m^6r^2a - 150m^2r^2a - 78m^{10}r^2a + 18n^{10}r^2a \\
& - 90n^8r^2a + 180n^6r^2a - 330m^8r^2a + 90n^2r^2a - 180n^4r^2a - 420m^4r^2a
\end{aligned}$$

$$\begin{aligned}
& - 324m^3n^4r^4 + 224m^{13}n^4r^4 + 108m^{15}n^4r^4 + 378m^9n^{10}r^4 - 280m^9n^8r^4 \\
& + 27m^3n^{16}r^4 + 108m^5n^{14}r^4 + 112m^7n^2r^4 + 112m^{15}n^2r^4 - 176m^3n^{14}r^4 \\
& + 220m^7n^8r^4 + 252m^{13}n^6r^4 + 378m^{11}n^8r^4 - 100m^9n^6r^4 - 448m^8n^8r^4 \\
& + 36m^{11}n^4r^4 + 252m^7n^{12}r^4 - 136m^4n^4r^4 - 72m^9n^4r^4 + 308m^6n^6r^4
\end{aligned}$$

$$\begin{aligned}
& + 612m^5n^{10}r^4 + 112m^{11}n^6r^4 + 140m^{13}n^2r^4 - 560m^7n^{10}r^4 + 160m^3n^2r^4 \\
& + 252m^5n^2r^4 - 216m^5n^8r^4 - 576m^3n^{10}r^4 + 35m^2n^2r^4 - 468m^2n^4r^2a \\
& - 294m^8n^2r^2a - 88m^5r^4 - 112m^9r^4 - 14m^{11}r^4 + 348m^4n^2r^2a - 396m^6n^4r^2a \\
& - 140m^7r^4 - 6m^2n^8r^2a - 312m^6n^2r^2a + 3m^{19}r^4 - 29m^3r^4 + 56m^{13}r^4
\end{aligned}$$

$$\begin{aligned}
& + 52m^{15}r^4 - 448m^5n^{12}r^4 + 20m^{17}r^4 + 3m^{18}r^4 + 116m^{14}r^4 - 28m^{16}r^4 \\
& - 280m^{12}r^4 + 434m^{10}r^4 + 52m^7n^4r^4 + 456m^2n^2r^2a + 276m^4n^4r^2a \\
& + 168m^2n^6r^2a - 204m^4n^6r^2a + 64m^7n^6r^4 - 70m^9n^2r^4 - 76m^5n^6r^4
\end{aligned}$$

$$\begin{aligned}
& + 288a^2m)/m, G02 = \frac{1}{432}r^2(-100m^4n^8r^2 + 468m^6n^4r^2 - 460m^4n^6r^2 \\
& - 1368m^6n^2r^2 + 490m^2n^8r^2 + 616m^2n^{10}r^2 + 112m^6n^6r^2 - 550m^8n^2r^2 \\
& - 148m^8n^4r^2 - 812m^2n^{12}r^2 + 612m^{12}n^2r^2 - 468m^4n^{10}r^2 + 36m^6n^8r^2
\end{aligned}$$

$$\begin{aligned}
& - 76 m^8 n^6 r^2 + 424 m^{10} n^2 r^2 + 288 m^3 n^2 a - 488 m^2 n^2 r^2 + 252 m^{10} n^4 r^2 \\
& - 432 n^4 a m + 288 n^2 a m + 1476 m^4 n^4 r^2 + 994 m^{10} r^2 - 324 n^{14} r^2 + 268 m^{14} r^2 \\
& - 1134 n^{10} r^2 + 644 m^{12} r^2 + 756 n^{12} r^2 + 1204 m^2 n^4 r^2 - 1400 m^2 n^6 r^2 - 72 a m \\
& - 288 m^3 a - 432 m^5 a - 1180 m^4 n^2 r^2 + 1022 m^8 r^2 + 700 m^6 r^2 + 308 m^4 r^2
\end{aligned}$$

$$\begin{aligned}
& + 79 m^2 r^2 + 324 n^4 r^2 - 81 n^2 r^2 + 1134 n^8 r^2 - 756 n^6 r^2 + 9 r^2 + 7 m^{18} r^2 \\
& + 136 r^2 n^6 m^{10} + 360 r^2 n^{10} m^6 - 10 r^2 n^8 m^8 + 620 r^2 n^{12} m^4 + 280 r^2 n^2 m^{14} \\
& + 396 r^2 n^4 m^{12} + 376 r^2 n^{14} m^2 - 9 n^{18} r^2 + 65 r^2 m^{16} + 81 r^2 n^{16} - 288 m^3 n^6 a \\
& + 288 m n^6 a - 72 m n^8 a - 288 m^7 n^2 a - 288 m^5 n^2 a - 432 m^5 n^4 a + 288 m^3 n^4 a
\end{aligned}$$

$$\begin{aligned}
& + 140 m^{12} n^6 r^2 + 124 m^{14} n^4 r^2 - 14 m^{10} n^8 r^2 - 65 m^2 n^{16} r^2 - 308 m^6 n^{12} r^2 \\
& - 196 m^4 n^{14} r^2 + 47 m^{16} n^2 r^2 - 238 m^8 n^{10} r^2 - 72 m^9 a - 288 m^7 a)/m, a = a, r = r,
\end{aligned}$$

$$m = m, n = n, h201 = - 15 r^5 n^{14} m^3 - \frac{55}{18} r^5 m^{19} n^6 + \frac{5}{3} r^5 n^6 m^{11} + \frac{7}{3} r^5 n^4 m^{11}$$

$$\begin{aligned}
& - \frac{77}{6} r^5 m^{13} n^{12} - \frac{5}{3} r^5 n^6 m^9 - \frac{1}{6} r^5 n^{22} m^3 + 7 r^5 n^{12} m^3 - \frac{35}{6} r^5 n^{18} m^3 - 15 r^5 n^2 m^{15} \\
& - \frac{7}{3} r^5 n^{10} m^5 + \frac{5}{3} r^5 n^8 m^{11} - \frac{35}{6} r^5 n^{12} m^5 - \frac{25}{2} r^5 n^2 m^{17} + \frac{50}{3} r^5 n^{14} m^5 + \frac{5}{3} r^5 n^{10} m^7
\end{aligned}$$

$$\begin{aligned}
& + \frac{5}{3} r^5 n^8 m^7 + \frac{50}{9} r^5 n^{12} m^7 + \frac{25}{2} r^5 n^{16} m^3 - \frac{5}{4} r^5 n^8 m^9 - \frac{35}{6} r^5 m^{19} n^2 - \frac{175}{12} r^5 n^4 m^{17} \\
& - \frac{175}{12} r^5 n^{16} m^5 - \frac{50}{3} r^5 n^6 m^{15} - \frac{35}{6} r^5 m^{19} n^4 - \frac{55}{8} r^5 n^8 m^{17} - \frac{25}{2} r^5 n^6 m^{17}
\end{aligned}$$

$$\begin{aligned}
& - \frac{50}{3} r^5 n^{14} m^7 + \frac{35}{6} r^5 n^{18} m^5 - 15 r^5 n^8 m^{15} - \frac{5}{3} r^5 n^{10} m^9 + \frac{55}{18} m n^{18} r^5 - \frac{1}{72} m n^{24} r^5 \\
& + \frac{3}{2} r^5 n^{20} m^3 - 7 r^5 m^{13} n^{10} - 11 r^5 n^{10} m^{15} + \frac{55}{18} m n^6 r^5 + \frac{1}{6} m n^2 r^5 - \frac{11}{12} m n^4 r^5
\end{aligned}$$

$$\begin{aligned}
& - \frac{11}{12} m n^{20} r^5 + 15 r^5 n^2 m^9 + \frac{25}{2} r^5 n^2 m^7 + \frac{35}{6} r^5 n^2 m^5 + \frac{25}{2} r^5 n^6 m^3 - \frac{35}{6} r^5 n^4 m^3 \\
& + 7 r^5 n^{10} m^3 - 15 r^5 n^8 m^3 - 11 r^5 n^{14} m^{11} + \frac{50}{9} r^5 n^6 m^7 + \frac{50}{3} r^5 n^6 m^5 - \frac{50}{3} r^5 n^4 m^7
\end{aligned}$$

$$\begin{aligned}
& -\frac{35}{6}r^5n^4m^9 + \frac{3}{2}r^5n^2m^3 + 7r^5n^2m^{11} + 7r^5n^{12}m^{11} + \frac{7}{3}r^5n^{10}m^{11} - \frac{55}{18}r^5n^{18}m^7 \\
& -\frac{11}{12}r^5n^{20}m^5 - \frac{55}{8}r^5n^{16}m^9 - 7r^5m^{13}n^2 - \frac{35}{6}r^5n^8m^{13} - \frac{50}{9}r^5m^{13}n^6 - \frac{35}{6}r^5m^{13}n^4 \\
& -\frac{175}{12}r^5n^4m^5 - \frac{35}{6}r^5n^8m^5 - \frac{1}{6}r^5m^{23}n^2 - \frac{11}{12}r^5n^4m^{21} + 15r^5n^{14}m^9 + \frac{25}{2}r^5n^{16}m^7 \\
& -\frac{3}{2}r^5n^2m^{21} + 11mn^{10}r^5 - \frac{55}{8}mn^8r^5 - \frac{55}{8}mn^{16}r^5 + 11mn^{14}r^5 - \frac{77}{6}mn^{12}r^5 \\
& +\frac{1}{6}mn^{22}r^5 - \frac{1}{72}r^5m^{25} - \frac{11}{12}r^5m^{21} - \frac{55}{18}r^5m^7 - \frac{1}{6}r^5m^{23} - 11r^5m^{15} - \frac{77}{6}r^5m^{13} \\
& -\frac{11}{12}r^5m^5 - \frac{1}{6}r^5m^3 - 11r^5m^{11} - \frac{55}{8}r^5m^9 - \frac{55}{8}r^5m^{17} - \frac{1}{72}mr^5 - \frac{55}{18}r^5m^{19} \\
& -\frac{50}{3}r^5n^4m^{15} - \frac{35}{6}r^5n^{12}m^9 - \frac{14}{3}r^3m^{12}an^4 - \frac{28}{3}r^3m^{10}an^6 - \frac{8}{3}r^3n^6m^4a \\
& -\frac{35}{3}r^3m^8an^8 + 12r^3am^4n^{10} - 6r^3n^8m^4a + \frac{8}{3}r^3m^6an^6 + \frac{20}{3}r^3am^6n^8 \\
& -\frac{20}{3}r^3m^{12}an^2 + \frac{8}{3}r^3m^6an^4 + \frac{20}{3}r^3am^6n^2 - \frac{20}{3}r^3m^8an^2 - 6r^3m^8an^4 \\
& -12r^3m^{10}an^2 - 6r^3am^4n^4 - \frac{1}{6}r^3m^{16}a - \frac{4}{3}r^3m^{14}a - \frac{35}{3}r^3am^8 - \frac{1}{6}n^{16}r^3a \\
& -\frac{28}{3}m^{10}r^3a - \frac{14}{3}r^3m^{12}a + \frac{28}{3}n^{10}r^3a + \frac{4}{3}n^2r^3a - \frac{14}{3}n^4r^3a - \frac{14}{3}r^3am^4 \\
& -\frac{35}{3}n^8r^3a - \frac{14}{3}n^{12}r^3a + \frac{4}{3}n^{14}r^3a + \frac{28}{3}n^6r^3a - \frac{28}{3}r^3m^6a - \frac{4}{3}m^2r^3a - \frac{1}{6}r^3a \\
& -\frac{4}{3}m^2r^3n^{14}a + \frac{20}{3}m^2r^3an^6 + \frac{20}{3}m^2r^3n^{12}a + 12r^3n^2m^4a + \frac{20}{3}m^2n^8r^3a \\
& -12m^2r^3n^{10}a - 12m^2r^3n^4a + \frac{20}{3}m^2r^3an^2 - \frac{20}{3}r^3m^8an^6 - 12r^3m^{10}an^4
\end{aligned}$$

$$-\frac{4}{3}r^3 m^{14} a n^2 - \frac{28}{3}r^3 n^{10} a m^6 - \frac{14}{3}r^3 n^{12} a m^4, f11 = 0, g21 = \frac{1}{144}r^2 (-140 m^4 n^8 r^2$$

$$+ 252 m^6 n^4 r^2 - 260 m^4 n^6 r^2 - 648 m^6 n^2 r^2 + 350 m^2 n^8 r^2 + 56 m^2 n^{10} r^2$$

$$+ 80 m^6 n^6 r^2 - 290 m^8 n^2 r^2 - 92 m^8 n^4 r^2 - 196 m^2 n^{12} r^2 + 396 m^{12} n^2 r^2$$

$$+ 36 m^4 n^{10} r^2 + 108 m^6 n^8 r^2 - 68 m^8 n^6 r^2 + 248 m^{10} n^2 r^2 - 288 m^3 n^2 a$$

$$- 184 m^2 n^2 r^2 + 180 m^{10} n^4 r^2 + 432 n^4 a m - 288 n^2 a m + 684 m^4 n^4 r^2 + 518 m^{10} r^2$$

$$- 108 n^{14} r^2 + 164 m^{14} r^2 - 378 n^{10} r^2 + 364 m^{12} r^2 + 252 n^{12} r^2 + 476 m^2 n^4 r^2$$

$$- 616 m^2 n^6 r^2 + 72 a m + 288 m^3 a + 432 m^5 a - 500 m^4 n^2 r^2 + 490 m^8 r^2$$

$$+ 308 m^6 r^2 + 124 m^4 r^2 + 29 m^2 r^2 + 108 n^4 r^2 - 27 n^2 r^2 + 378 n^8 r^2 - 252 n^6 r^2$$

$$+ 3 r^2 + 5 m^{18} r^2 + 152 r^2 n^6 m^{10} - 72 r^2 n^{10} m^6 - 110 r^2 n^8 m^8 + 100 r^2 n^{12} m^4$$

$$+ 200 r^2 n^2 m^{14} + 324 r^2 n^4 m^{12} + 104 r^2 n^{14} m^2 - 3 n^{18} r^2 + 43 r^2 m^{16} + 27 r^2 n^{16}$$

$$+ 288 m^3 n^6 a - 288 m n^6 a + 72 m n^8 a + 288 m^7 n^2 a + 288 m^5 n^2 a + 432 m^5 n^4 a$$

$$- 288 m^3 n^4 a + 196 m^{12} n^6 r^2 + 116 m^{14} n^4 r^2 + 182 m^{10} n^8 r^2 - 19 m^2 n^{16} r^2$$

$$- 28 m^6 n^{12} r^2 - 44 m^4 n^{14} r^2 + 37 m^{16} n^2 r^2 + 70 m^8 n^{10} r^2 + 72 m^9 a + 288 m^7 a)/m,$$

$$F3 = \frac{1}{144} (-192 m^3 r^2 a - 268 m^8 n^6 r^4 - 180 m^{10} n^4 r^4 + 160 m^{14} n^2 r^4 + 64 m^2 n^{14} r^4$$

$$- 340 m^4 n^6 r^4 + 196 m^{12} n^2 r^4 - 490 m^8 n^2 r^4 + 196 m^{12} n^6 r^4 + 504 m^4 n^4 r^4$$

$$+ 116 m^{14} n^4 r^4 - 48 m r^2 a - 448 m^6 n^2 r^4 + 182 m^{10} n^8 r^4 - 112 m^{10} n^2 r^4$$

$$- 460 m^8 n^8 r^4 + 308 m^6 n^8 r^4 + 184 m^{12} n^4 r^4 - 320 m^4 n^8 r^4 + 550 m^2 n^8 r^4$$

$$- 352 m^6 n^{10} r^4 - 19 m^2 n^{16} r^4 - 128 m^{10} n^6 r^4 + 396 m^4 n^{10} r^4 - 28 m^6 n^{12} r^4$$

$$+ 4 m^2 n^{12} r^4 + 112 n^{12} r^4 - 98 n^{10} r^4 + 28 n^8 r^4 + 28 n^6 r^4 - 32 n^4 r^4 + 5 m^{18} r^4$$

$$+ 38 m^{16} r^4 + 124 m^{14} r^4 + 224 m^{12} r^4 + 238 m^{10} r^4 + 140 m^8 r^4 + 28 m^6 r^4 - 16 m^4 r^4$$

$$- 11 m^2 r^4 - 3 n^{18} r^4 + 22 n^{16} r^4 + 13 n^2 r^4 - 68 n^{14} r^4 - 416 m^2 n^6 r^4 + 116 m^2 n^4 r^4$$

$$+ 16 m^2 n^2 r^4 + 160 m^6 n^6 r^4 - 44 m^4 n^{14} r^4 - 140 m^4 n^2 r^4 + 332 m^6 n^4 r^4$$

$$\begin{aligned}
& - 272 m^8 n^4 r^4 - 304 m^2 n^{10} r^4 + 37 m^{16} n^2 r^4 + 70 m^8 n^{10} r^4 - 40 m^4 n^{12} r^4 - 2 r^4 \\
& - 288 m^5 r^2 a - 48 m^9 r^2 a - 192 m^7 r^2 a + 192 m^3 n^2 r^2 a + 192 m n^2 r^2 a \\
& - 288 m n^4 r^2 a - 192 m^7 n^2 r^2 a - 192 m^5 n^2 r^2 a - 288 m^5 n^4 r^2 a + 192 m^3 n^4 r^2 a \\
& - 192 m^3 n^6 r^2 a + 192 m n^6 r^2 a - 48 m n^8 r^2 a + 144 a^2) / (
\end{aligned}$$

$$r(m^2 + 1 - 2n + n^2)(m^2 + 1 + 2n + n^2)), f_3 = -\frac{1}{6} n^4 r a - \frac{1}{6} r a - \frac{1}{3} m^2 n^2 r a$$

$$-\frac{1}{3} m^2 r a + \frac{1}{3} n^2 r a - \frac{1}{6} m^4 r a + \frac{5}{6} r^3 m^7 - \frac{5}{6} m n^6 r^3 - \frac{1}{4} m n^2 r^3 + \frac{5}{8} m n^4 r^3$$

$$+ \frac{3}{4} r^3 n^2 m^9 + \frac{1}{2} r^3 n^2 m^7 - \frac{1}{2} r^3 n^2 m^5 + \frac{1}{2} r^3 n^6 m^3 + \frac{1}{2} r^3 n^4 m^3 + \frac{1}{4} r^3 n^{10} m^3$$

$$- \frac{3}{4} r^3 n^8 m^3 + \frac{5}{6} r^3 n^6 m^7 - \frac{1}{2} r^3 n^6 m^5 + \frac{1}{2} r^3 n^4 m^7 + \frac{5}{8} r^3 n^4 m^9 - \frac{3}{4} r^3 n^2 m^3$$

$$+ \frac{1}{4} r^3 n^2 m^{11} - \frac{1}{4} r^3 n^4 m^5 + \frac{5}{8} r^3 n^8 m^5 - \frac{1}{4} m n^{10} r^3 + \frac{5}{8} m n^8 r^3 + \frac{1}{24} m n^{12} r^3 + \frac{1}{4} r^3 m^{11}$$

$$+ \frac{5}{8} r^3 m^9 + \frac{1}{24} m r^3 + \frac{1}{4} r^3 m^3 + \frac{1}{24} r^3 m^{13} + \frac{5}{8} r^3 m^5, H_{201} = -\frac{1}{144} r (228 m^8 n^6 r^4$$

$$+ 540 m^{10} n^4 r^4 + 96 m^{14} n^2 r^4 - 384 m^2 n^{14} r^4 + 252 m^4 n^6 r^4 + 244 m^{12} n^2 r^4$$

$$+ 510 m^8 n^2 r^4 + 308 m^{12} n^6 r^4 - 648 m^4 n^4 r^4 + 100 m^{14} n^4 r^4 + 512 m^6 n^2 r^4$$

$$+ 574 m^{10} n^8 r^4 + 400 m^{10} n^2 r^4 + 180 m^8 n^8 r^4 - 28 m^6 n^8 r^4 + 376 m^{12} n^4 r^4$$

$$+ 32 m^4 n^8 r^4 - 210 m^2 n^8 r^4 - 544 m^6 n^{10} r^4 + 73 m^2 n^{16} r^4 + 576 m^{10} n^6 r^4$$

$$+ 540 m^4 n^{10} r^4 + 532 m^6 n^{12} r^4 + 756 m^2 n^{12} r^4 - 560 n^{12} r^4 + 742 n^{10} r^4 - 644 n^8 r^4$$

$$+ 364 n^6 r^4 - 128 n^4 r^4 + m^{18} r^4 + 6 m^{16} r^4 + 12 m^{14} r^4 - 42 m^{10} r^4 - 84 m^8 r^4$$

$$- 84 m^6 r^4 - 48 m^4 r^4 - 15 m^2 r^4 + 9 n^{18} r^4 - 74 n^{16} r^4 + 25 n^2 r^4 + 268 n^{14} r^4$$

$$+ 672 m^2 n^6 r^4 - 476 m^2 n^4 r^4 + 144 m^2 n^2 r^4 - 96 m^6 n^6 r^4 + 260 m^4 n^{14} r^4$$

$$+ 356 m^4 n^2 r^4 - 292 m^6 n^4 r^4 + 272 m^8 n^4 r^4 - 560 m^2 n^{10} r^4 + 17 m^{16} n^2 r^4$$

$$\begin{aligned}
& + 686 m^8 n^{10} r^4 - 744 m^4 n^{12} r^4 - 2 r^4 + 144 a^2) (m^8 + 4 m^6 n^2 + 4 m^6 + 4 m^4 n^2 \\
& + 6 m^4 + 6 m^4 n^4 - 4 m^2 n^4 - 4 m^2 n^2 + 4 m^2 + 4 m^2 n^6 - 4 n^2 - 4 n^6 + 1 + n^8 + 6 n^4), \\
HO11 = & - 30 r^5 n^{14} m^3 - \frac{55}{9} r^5 m^{19} n^6 + \frac{10}{3} r^5 n^6 m^{11} + \frac{14}{3} r^5 n^4 m^{11} - \frac{77}{3} r^5 m^{13} n^{12} \\
& - \frac{10}{3} r^5 n^6 m^9 - \frac{1}{3} r^5 n^{22} m^3 + 14 r^5 n^{12} m^3 - \frac{35}{3} r^5 n^{18} m^3 - 30 r^5 n^2 m^{15} - \frac{14}{3} r^5 n^{10} m^5 \\
& + \frac{10}{3} r^5 n^8 m^{11} - \frac{35}{3} r^5 n^{12} m^5 - 25 r^5 n^2 m^{17} + \frac{100}{3} r^5 n^{14} m^5 + \frac{10}{3} r^5 n^{10} m^7 \\
& + \frac{10}{3} r^5 n^8 m^7 + \frac{100}{9} r^5 n^{12} m^7 + 25 r^5 n^{16} m^3 - \frac{5}{2} r^5 n^8 m^9 - \frac{35}{3} r^5 m^{19} n^2 \\
& - \frac{175}{6} r^5 n^4 m^{17} - \frac{175}{6} r^5 n^{16} m^5 - \frac{100}{3} r^5 n^6 m^{15} - \frac{35}{3} r^5 m^{19} n^4 - \frac{55}{4} r^5 n^8 m^{17} \\
& - 25 r^5 n^6 m^{17} - \frac{100}{3} r^5 n^{14} m^7 + \frac{35}{3} r^5 n^{18} m^5 - 30 r^5 n^8 m^{15} - \frac{10}{3} r^5 n^{10} m^9 \\
& + \frac{55}{9} m n^{18} r^5 - \frac{1}{36} m n^{24} r^5 + 3 r^5 n^{20} m^3 - 14 r^5 m^{13} n^{10} - 22 r^5 n^{10} m^{15} + \frac{55}{9} m n^6 r^5 \\
& + \frac{1}{3} m n^2 r^5 - \frac{11}{6} m n^4 r^5 - \frac{11}{6} m n^{20} r^5 + 30 r^5 n^2 m^9 + 25 r^5 n^2 m^7 + \frac{35}{3} r^5 n^2 m^5 \\
& + 25 r^5 n^6 m^3 - \frac{35}{3} r^5 n^4 m^3 + 14 r^5 n^{10} m^3 - 30 r^5 n^8 m^3 - 22 r^5 n^{14} m^{11} + \frac{100}{9} r^5 n^6 m^7 \\
& + \frac{100}{3} r^5 n^6 m^5 - \frac{100}{3} r^5 n^4 m^7 - \frac{35}{3} r^5 n^4 m^9 + 3 r^5 n^2 m^3 + 14 r^5 n^2 m^{11} + 14 r^5 n^{12} m^{11} \\
& + \frac{14}{3} r^5 n^{10} m^{11} - \frac{55}{9} r^5 n^{18} m^7 - \frac{11}{6} r^5 n^{20} m^5 - \frac{55}{4} r^5 n^{16} m^9 - 14 r^5 m^{13} n^2 \\
& - \frac{35}{3} r^5 n^8 m^{13} - \frac{100}{9} r^5 m^{13} n^6 - \frac{35}{3} r^5 m^{13} n^4 - \frac{175}{6} r^5 n^4 m^5 - \frac{35}{3} r^5 n^8 m^5 \\
& - \frac{1}{3} r^5 m^{23} n^2 - \frac{11}{6} r^5 n^4 m^{21} + 30 r^5 n^{14} m^9 + 25 r^5 n^{16} m^7 - 3 r^5 n^2 m^{21} + 22 m n^{10} r^5
\end{aligned}$$



$$\begin{aligned}
& -\frac{55}{4} m n^8 r^5 - \frac{55}{4} m n^{16} r^5 + 22 m n^{14} r^5 - \frac{77}{3} m n^{12} r^5 + \frac{1}{3} m n^{22} r^5 - \frac{1}{36} r^5 m^{25} \\
& - \frac{11}{6} r^5 m^{21} - \frac{55}{9} r^5 m^7 - \frac{1}{3} r^5 m^{23} - 22 r^5 m^{15} - \frac{77}{3} r^5 m^{13} - \frac{11}{6} r^5 m^5 - \frac{1}{3} r^5 m^3 \\
& - 22 r^5 m^{11} - \frac{55}{4} r^5 m^9 - \frac{55}{4} r^5 m^{17} - \frac{1}{36} m r^5 - \frac{55}{9} r^5 m^{19} - \frac{100}{3} r^5 n^4 m^{15} \\
& - \frac{35}{3} r^5 n^{12} m^9 - \frac{28}{3} r^3 m^{12} a n^4 - \frac{56}{3} r^3 m^{10} a n^6 - \frac{16}{3} r^3 n^6 m^4 a - \frac{70}{3} r^3 m^8 a n^8 \\
& + 24 r^3 a m^4 n^{10} - 12 r^3 n^8 m^4 a + \frac{16}{3} r^3 m^6 a n^6 + \frac{40}{3} r^3 a m^6 n^8 - \frac{40}{3} r^3 m^{12} a n^2 \\
& + \frac{16}{3} r^3 m^6 a n^4 + \frac{40}{3} r^3 a m^6 n^2 - \frac{40}{3} r^3 m^8 a n^2 - 12 r^3 m^8 a n^4 - 24 r^3 m^{10} a n^2 \\
& - 12 r^3 a m^4 n^4 - \frac{1}{3} r^3 m^{16} a - \frac{8}{3} r^3 m^{14} a - \frac{70}{3} r^3 a m^8 - \frac{1}{3} n^{16} r^3 a - \frac{56}{3} m^{10} r^3 a \\
& - \frac{28}{3} r^3 m^{12} a + \frac{56}{3} n^{10} r^3 a + \frac{8}{3} n^2 r^3 a - \frac{28}{3} n^4 r^3 a - \frac{28}{3} r^3 a m^4 - \frac{70}{3} n^8 r^3 a \\
& - \frac{28}{3} n^{12} r^3 a + \frac{8}{3} n^{14} r^3 a + \frac{56}{3} n^6 r^3 a - \frac{56}{3} r^3 m^6 a - \frac{8}{3} m^2 r^3 a - \frac{1}{3} r^3 a - \frac{8}{3} m^2 r^3 n^{14} a \\
& + \frac{40}{3} m^2 r^3 a n^6 + \frac{40}{3} m^2 r^3 n^{12} a + 24 r^3 n^2 m^4 a + \frac{40}{3} m^2 n^8 r^3 a - 24 m^2 r^3 n^{10} a \\
& - 24 m^2 r^3 n^4 a + \frac{40}{3} m^2 r^3 a n^2 - \frac{40}{3} r^3 m^8 a n^6 - 24 r^3 m^{10} a n^4 - \frac{8}{3} r^3 m^{14} a n^2 \\
& - \frac{56}{3} r^3 n^{10} a m^6 - \frac{28}{3} r^3 n^{12} a m^4, F001 = -\frac{1}{12} n^4 r a - \frac{1}{12} r a - \frac{1}{6} m^2 n^2 r a - \frac{1}{6} m^2 r a \\
& + \frac{1}{6} n^2 r a - \frac{1}{12} m^4 r a - \frac{5}{36} r^3 m^7 + \frac{5}{36} m n^6 r^3 + \frac{1}{24} m n^2 r^3 - \frac{5}{48} m n^4 r^3 - \frac{1}{8} r^3 n^2 m^9 \\
& - \frac{1}{12} r^3 n^2 m^7 + \frac{1}{12} r^3 n^2 m^5 - \frac{1}{12} r^3 n^6 m^3 - \frac{1}{12} r^3 n^4 m^3 - \frac{1}{24} r^3 n^{10} m^3 + \frac{1}{8} r^3 n^8 m^3
\end{aligned}$$

$$\begin{aligned}
& -\frac{5}{36}r^3 n^6 m^7 + \frac{1}{12}r^3 n^6 m^5 - \frac{1}{12}r^3 n^4 m^7 - \frac{5}{48}r^3 n^4 m^9 + \frac{1}{8}r^3 n^2 m^3 - \frac{1}{24}r^3 n^2 m^{11} \\
& + \frac{1}{24}r^3 n^4 m^5 - \frac{5}{48}r^3 n^8 m^5 + \frac{1}{24}m n^{10} r^3 - \frac{5}{48}m n^8 r^3 - \frac{1}{144}m n^{12} r^3 - \frac{1}{24}r^3 m^{11} \\
& - \frac{5}{48}r^3 m^9 - \frac{1}{144}m r^3 - \frac{1}{24}r^3 m^3 - \frac{1}{144}r^3 m^{13} - \frac{5}{48}r^3 m^5, F11 = \frac{1}{3}n^4 r a + \frac{1}{3}r a \\
& + \frac{2}{3}m^2 n^2 r a + \frac{2}{3}m^2 r a - \frac{2}{3}n^2 r a + \frac{1}{3}m^4 r a - \frac{25}{27}r^3 m^7 + \frac{25}{27}m n^6 r^3 + \frac{5}{18}m n^2 r^3 \\
& - \frac{25}{36}m n^4 r^3 - \frac{5}{6}r^3 n^2 m^9 - \frac{5}{9}r^3 n^2 m^7 + \frac{5}{9}r^3 n^2 m^5 - \frac{5}{9}r^3 n^6 m^3 - \frac{5}{9}r^3 n^4 m^3 \\
& - \frac{5}{18}r^3 n^{10} m^3 + \frac{5}{6}r^3 n^8 m^3 - \frac{25}{27}r^3 n^6 m^7 + \frac{5}{9}r^3 n^6 m^5 - \frac{5}{9}r^3 n^4 m^7 - \frac{25}{36}r^3 n^4 m^9 \\
& + \frac{5}{6}r^3 n^2 m^3 - \frac{5}{18}r^3 n^2 m^{11} + \frac{5}{18}r^3 n^4 m^5 - \frac{25}{36}r^3 n^8 m^5 + \frac{5}{18}m n^{10} r^3 - \frac{25}{36}m n^8 r^3 \\
& - \frac{5}{108}m n^{12} r^3 - \frac{5}{18}r^3 m^{11} - \frac{25}{36}r^3 m^9 - \frac{5}{108}m r^3 - \frac{5}{18}r^3 m^3 - \frac{5}{108}r^3 m^{13} - \frac{25}{36}r^3 m^5, \\
C2 = & -\frac{1}{54}m^9 r^2 - \frac{2}{27}m^7 n^2 r^2 - \frac{2}{27}m^7 r^2 - \frac{1}{9}m^5 r^2 - \frac{1}{9}m^5 n^4 r^2 - \frac{2}{27}m^5 n^2 r^2 \\
& + \frac{2}{27}m^3 n^2 r^2 + \frac{2}{27}m^3 n^4 r^2 - \frac{2}{27}m^3 n^6 r^2 - \frac{2}{27}m^3 r^2 - \frac{1}{54}m r^2 + \frac{2}{27}m n^2 r^2 - \frac{1}{9}m n^4 r^2 \\
& + \frac{2}{27}m n^6 r^2 - \frac{1}{54}m n^8 r^2, G21 = -\frac{1}{36}(m r^4 - 9 r^2 a + 36 m^5 n^4 r^4 - 40 m^3 n^8 r^4 \\
& - 40 m^3 n^6 r^4 - 40 m^3 n^{12} r^4 - 126 m^6 r^2 a - 51 m^2 r^2 a - 15 m^{10} r^2 a + 9 n^{10} r^2 a \\
& - 45 n^8 r^2 a + 90 n^6 r^2 a - 69 m^8 r^2 a + 45 n^2 r^2 a - 90 n^4 r^2 a - 114 m^4 r^2 a \\
& + 72 m^3 n^4 r^4 + 28 m^{13} n^4 r^4 + 70 m^9 n^8 r^4 - 40 m^7 n^2 r^4 + 8 m^{15} n^2 r^4 + 8 m^3 n^{14} r^4 \\
& - 40 m^7 n^8 r^4 + 40 m^9 n^6 r^4 + 70 m n^8 r^4 + 72 m^{11} n^4 r^4 + 28 m n^4 r^4 + 36 m^9 n^4 r^4 \\
& - 56 m n^6 r^4 - 72 m^5 n^{10} r^4 + 56 m^{11} n^6 r^4 + 40 m^{13} n^2 r^4 + 56 m^7 n^{10} r^4 - 40 m^3 n^2 r^4 \\
& - 72 m^5 n^2 r^4 + 72 m^{11} n^2 r^4 + 36 m^5 n^8 r^4 + 72 m^3 n^{10} r^4 - 8 m n^2 r^4 - 90 m^2 n^4 r^2 a
\end{aligned}$$

$$\begin{aligned}
& - 51 m^8 n^2 r^2 a + 28 m^5 r^4 + 70 m^9 r^4 + 56 m^{11} r^4 + 78 m^4 n^2 r^2 a - 54 m^6 n^4 r^2 a \\
& + 56 m^7 r^4 + 21 m^2 n^8 r^2 a - 60 m^6 n^2 r^2 a + 8 m^3 r^4 + 28 m^{13} r^4 + 8 m^{15} r^4 \\
& + 28 m^5 n^{12} r^4 + m^{17} r^4 - 8 m n^{14} r^4 + m n^{16} r^4 + 28 m n^{12} r^4 - 56 m n^{10} r^4 \\
& - 16 m^7 n^4 r^4 + 132 m^2 n^2 r^2 a + 42 m^4 n^4 r^2 a - 12 m^2 n^6 r^2 a - 6 m^4 n^6 r^2 a
\end{aligned}$$

$$- 16 m^7 n^6 r^4 + 40 m^9 n^2 r^4 + 16 m^5 n^6 r^4 - 72 a^2 m) / m, G101 = - \frac{19}{12} m^8 n^6 r^4$$

$$- \frac{15}{4} m^{10} n^4 r^4 - \frac{2}{3} m^{14} n^2 r^4 + \frac{8}{3} m^2 n^{14} r^4 - \frac{7}{4} m^4 n^6 r^4 - \frac{61}{36} m^{12} n^2 r^4 - \frac{85}{24} m^8 n^2 r^4$$

$$\begin{aligned}
& - \frac{77}{36} m^{12} n^6 r^4 + \frac{9}{2} m^4 n^4 r^4 - \frac{25}{36} m^{14} n^4 r^4 - \frac{32}{9} m^6 n^2 r^4 - \frac{287}{72} m^{10} n^8 r^4 - \frac{25}{9} m^{10} n^2 r^4 \\
& - \frac{5}{4} m^8 n^8 r^4 + \frac{7}{36} m^6 n^8 r^4 - \frac{47}{18} m^{12} n^4 r^4 - \frac{2}{9} m^4 n^8 r^4 + \frac{35}{24} m^2 n^8 r^4 + \frac{34}{9} m^6 n^{10} r^4
\end{aligned}$$

$$\begin{aligned}
& - \frac{73}{144} m^2 n^{16} r^4 - 4 m^{10} n^6 r^4 - \frac{15}{4} m^4 n^{10} r^4 - \frac{133}{36} m^6 n^{12} r^4 - \frac{21}{4} m^2 n^{12} r^4 + \frac{35}{9} n^{12} r^4 \\
& - \frac{371}{72} n^{10} r^4 + \frac{161}{36} n^8 r^4 - \frac{91}{36} n^6 r^4 + \frac{8}{9} n^4 r^4 - \frac{1}{144} m^{18} r^4 - \frac{1}{24} m^{16} r^4 - \frac{1}{12} m^{14} r^4
\end{aligned}$$

$$\begin{aligned}
& + \frac{7}{24} m^{10} r^4 + \frac{7}{12} m^8 r^4 + \frac{7}{12} m^6 r^4 + \frac{1}{3} m^4 r^4 + \frac{5}{48} m^2 r^4 - \frac{1}{16} n^{18} r^4 + \frac{37}{72} n^{16} r^4 \\
& - \frac{25}{144} n^2 r^4 - \frac{67}{36} n^{14} r^4 - \frac{14}{3} m^2 n^6 r^4 + \frac{119}{36} m^2 n^4 r^4 - m^2 n^2 r^4 + \frac{2}{3} m^6 n^6 r^4
\end{aligned}$$

$$\begin{aligned}
& - \frac{65}{36} m^4 n^{14} r^4 - \frac{89}{36} m^4 n^2 r^4 + \frac{73}{36} m^6 n^4 r^4 - \frac{17}{9} m^8 n^4 r^4 + \frac{35}{9} m^2 n^{10} r^4 - \frac{17}{144} m^{16} n^2 r^4 \\
& - \frac{343}{72} m^8 n^{10} r^4 + \frac{31}{6} m^4 n^{12} r^4 + \frac{1}{72} r^4 - a^2, CI = \frac{1}{72} (-288 m^3 r^2 a + 68 m^8 n^6 r^4 \\
& + 252 m^{10} n^4 r^4 + 64 m^{14} n^2 r^4 - 416 m^2 n^{14} r^4 + 188 m^4 n^6 r^4 + 84 m^{12} n^2 r^4
\end{aligned}$$

$$\begin{aligned}
& + 350 m^8 n^2 r^4 + 308 m^{12} n^6 r^4 - 792 m^4 n^4 r^4 + 100 m^{14} n^4 r^4 - 72 m r^2 a \\
& + 672 m^6 n^2 r^4 + 574 m^{10} n^8 r^4 + 112 m^{10} n^2 r^4 - 100 m^8 n^8 r^4 + 132 m^6 n^8 r^4 \\
& + 264 m^{12} n^4 r^4 - 112 m^4 n^8 r^4 - 50 m^2 n^8 r^4 - 768 m^6 n^{10} r^4 + 73 m^2 n^{16} r^4 \\
& + 352 m^{10} n^6 r^4 + 828 m^4 n^{10} r^4 + 532 m^6 n^{12} r^4 + 916 m^2 n^{12} r^4 - 672 n^{12} r^4
\end{aligned}$$

$$\begin{aligned}
&+ 966 n^{10} r^4 - 924 n^8 r^4 + 588 n^6 r^4 - 240 n^4 r^4 + m^{18} r^4 + 2 m^{16} r^4 - 20 m^{14} r^4 \\
&- 112 m^{12} r^4 - 266 m^{10} r^4 - 364 m^8 r^4 - 308 m^6 r^4 - 160 m^4 r^4 - 47 m^2 r^4 + 9 n^{18} r^4 \\
&- 78 n^{16} r^4 + 57 n^2 r^4 + 300 n^{14} r^4 + 832 m^2 n^6 r^4 - 764 m^2 n^4 r^4 + 304 m^2 n^2 r^4 \\
&- 32 m^6 n^6 r^4 + 260 m^4 n^{14} r^4 + 644 m^4 n^2 r^4 - 228 m^6 n^4 r^4 + 128 m^8 n^4 r^4
\end{aligned}$$

$$\begin{aligned}
&- 848 m^2 n^{10} r^4 + 17 m^{16} n^2 r^4 + 686 m^8 n^{10} r^4 - 856 m^4 n^{12} r^4 - 6 r^4 - 432 m^5 r^2 a \\
&- 72 m^9 r^2 a - 288 m^7 r^2 a + 288 m^3 n^2 r^2 a + 288 m n^2 r^2 a - 432 m n^4 r^2 a \\
&- 288 m^7 n^2 r^2 a - 288 m^5 n^2 r^2 a - 432 m^5 n^4 r^2 a + 288 m^3 n^4 r^2 a - 288 m^3 n^6 r^2 a \\
&+ 288 m n^6 r^2 a - 72 m n^8 r^2 a + 432 a^2) / (
\end{aligned}$$

$$r^2 (m^2 + 1 - 2n + n^2)^2 (m^2 + 1 + 2n + n^2)^2 \}$$

>

```

CC1:=factor(1/72*(100*m^14*n^4*r^4+686*m^8*n^10*r^4+308*m^12*n^6*r^4-
4-32*m^6*n^6*r^4+574*m^10*n^8*r^4+532*m^6*n^12*r^4-
792*m^4*n^4*r^4+916*m^2*n^12*r^4-100*m^8*n^8*r^4-
50*m^2*n^8*r^4+672*m^6*n^2*r^4+352*m^10*n^6*r^4-
768*m^6*n^10*r^4+350*m^8*n^2*r^4-228*m^6*n^4*r^4+188*m^4*n^6*r^4-
416*m^2*n^14*r^4+84*m^12*n^2*r^4+17*m^16*n^2*r^4+832*m^2*n^6*r^4+13
2*m^6*n^8*r^4-924*n^8*r^4-
764*m^2*n^4*r^4+252*m^10*n^4*r^4+128*m^8*n^4*r^4+73*m^2*n^16*r^4-
848*m^2*n^10*r^4+264*m^12*n^4*r^4+64*m^14*n^2*r^4+68*m^8*n^6*r^4+82
8*m^4*n^10*r^4-6*r^4+588*n^6*r^4-240*n^4*r^4+m^18*r^4-20*m^14*r^4-
112*m^12*r^4-266*m^10*r^4-364*m^8*r^4-308*m^6*r^4-160*m^4*r^4-
47*m^2*r^4+9*n^18*r^4-
78*n^16*r^4+57*n^2*r^4+300*n^14*r^4+112*m^10*n^2*r^4+2*m^16*r^4-
672*n^12*r^4+966*n^10*r^4-
112*m^4*n^8*r^4+644*m^4*n^2*r^4+260*m^4*n^14*r^4+304*m^2*n^2*r^4+28
8*m^3*n^4*r^2*a+288*m*n^6*r^2*a-288*m^7*r^2*a-72*m^9*r^2*a-
432*m*n^4*r^2*a+288*m*n^2*r^2*a-288*m^3*r^2*a-72*m*r^2*a-
288*m^5*n^2*r^2*a-432*m^5*n^4*r^2*a-
432*m^5*r^2*a+288*m^3*n^2*r^2*a-288*m^3*n^6*r^2*a-72*m*n^8*r^2*a-
288*m^7*n^2*r^2*a-856*m^4*n^12*r^4+432*a^2)/(r^2*(m^2+1-
2*n+n^2)^2*(m^2+1+2*n+n^2)^2));
C2:=factor(-1/54*m^9*r^2-2/27*m^7*r^2-2/27*m^7*n^2*r^2-
1/9*m^5*n^4*r^2-1/9*m^5*r^2-
2/27*m^5*n^2*r^2+2/27*m^3*n^2*r^2+2/27*m^3*n^4*r^2-2/27*m^3*r^2-
2/27*m^3*n^6*r^2+2/27*m*n^6*r^2-1/9*m*n^4*r^2+2/27*m*n^2*r^2-
1/54*m*r^2-1/54*m*n^8*r^2);

```

$$\begin{aligned}
CCI := &\frac{1}{72} (-288 m^3 r^2 a + 288 m n^6 r^2 a - 72 m n^8 r^2 a - 432 m^5 n^4 r^2 a - 288 m^7 n^2 r^2 a \\
&- 432 m n^4 r^2 a - 32 m^6 n^6 r^4 - 288 m^3 n^6 r^2 a + 288 m n^2 r^2 a - 432 m^5 r^2 a \\
&+ 288 m^3 n^2 r^2 a + 352 m^{10} n^6 r^4 + 308 m^{12} n^6 r^4 - 6 r^4 + 966 n^{10} r^4 - 856 m^4 n^{12} r^4
\end{aligned}$$

$$\begin{aligned}
& + 828 m^4 n^{10} r^4 + 672 m^6 n^2 r^4 + 644 m^4 n^2 r^4 - 764 m^2 n^4 r^4 + 304 m^2 n^2 r^4 \\
& - 228 m^6 n^4 r^4 + 100 m^{14} n^4 r^4 + 252 m^{10} n^4 r^4 + 264 m^{12} n^4 r^4 + 188 m^4 n^6 r^4 \\
& + 916 m^2 n^{12} r^4 + 17 m^{16} n^2 r^4 - 416 m^2 n^{14} r^4 + 84 m^{12} n^2 r^4 + 574 m^{10} n^8 r^4 \\
& + 260 m^4 n^{14} r^4 + 64 m^{14} n^2 r^4 - 768 m^6 n^{10} r^4 + 68 m^8 n^6 r^4 + 132 m^6 n^8 r^4 \\
& - 672 n^{12} r^4 + 832 m^2 n^6 r^4 - 924 n^8 r^4 - 288 m^5 n^2 r^2 a + 288 m^3 n^4 r^2 a \\
& + 73 m^2 n^{16} r^4 + 432 a^2 + 128 m^8 n^4 r^4 + 350 m^8 n^2 r^4 - 78 n^{16} r^4 + 57 n^2 r^4 \\
& + 300 n^{14} r^4 - 72 m r^2 a - 160 m^4 r^4 - 47 m^2 r^4 + 112 m^{10} n^2 r^4 - 848 m^2 n^{10} r^4 \\
& - 240 n^4 r^4 + m^{18} r^4 + 2 m^{16} r^4 - 20 m^{14} r^4 - 112 m^{12} r^4 - 364 m^8 r^4 - 308 m^6 r^4 \\
& - 266 m^{10} r^4 + 9 n^{18} r^4 + 588 n^6 r^4 + 532 m^6 n^{12} r^4 - 288 m^7 r^2 a - 100 m^8 n^8 r^4 \\
& - 50 m^2 n^8 r^4 - 792 m^4 n^4 r^4 - 112 m^4 n^8 r^4 + 686 m^8 n^{10} r^4 - 72 m^9 r^2 a) / ( \\
& r^2 (m^2 + 1 - 2 n + n^2)^2 (m^2 + 1 + 2 n + n^2)^2) \\
& C2 := - \frac{1}{54} m r^2 (m^2 + 1 - 2 n + n^2)^2 (m^2 + 1 + 2 n + n^2)^2
\end{aligned}$$

Then we see that if m is non-zero, then C2 is also non-zero and it is not the cubic.  
>

## Type Va for alpha=0.

First step: (3,4)-normalization -----

(2,3,4) jet

> restart:

```

readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(x)*cos(n*y):
T:=exp(x)*sin(n*y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],9):
S2:=int(diff(int(diff(4*S1/(1-n^2),z),z),cz),cz):
T2:=int(diff(int(diff(16*T1,z),z),cz),cz):
S3:=S2:T3:=(T2/n/(n^2-1)):
Result check.
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
S[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z,cz))))/2;
S[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));

```

```
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=factor(expand(subs([z=0,cz=0],diff(T3,z,z,cz))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

$$S_{2,1,0,0} := \frac{1}{4}$$

$$S_{3,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$

$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := I$$

$$T_{3,0,0,0} := 0$$

**Introduce the notations.**

```
> k:=I:ck:=-I: M:=k*ck:
```

**Introduce new variables**

```
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
```

```
w3:=u3+I*(z^2*cz+cz^2*z):
```

```
cw3:=u3-I*(z^2*cz+cz^2*z):
```

**Write a mapping**

```
> f:=
```

```
r*ck*z+
```

```
(F2+I*f2)*z^2+(F01+I*f01)*w2:
```

```
cf:=
```

```
r*k*cz+
```

```
(F2-I*f2)*cz^2+(F01-I*f01)*cw2:
```

```
g:=
```

```
r^2*M*w2+
```

```
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3:
```

```
cg:=
```

```
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3:
```

```
h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3:
```

```
ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3:
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (3,4)-jet.**

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):
```

**Write the equations and the variables.**

```

> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,

Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:
> sol:=solve(EQ)[2];
sol:={G3=0,h4=0,H4=0,g3=0,g001=0,H101=0,h101=0,F2=1/12*r^2,
f01=1/12*r^2,F01=-f2,g11=-1/6*r^3,G001=r*f2,H21=-2*r^2*f2,h21=-1/6*r^4,
h02=-2*r^2*f2,r=r,n=n,f2=f2,G11=-2*r*f2,H02=-1/6*r^4}

```

## Second Step: (4,5)-normalization ----- (3,4,5) jet

```

> restart:
readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):
S:=exp(x)*cos(n*y):
T:=exp(x)*sin(n*y):
S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],9):
S2:=int(diff(int(diff(4*S1/(1-n^2),z),z),cz),cz):
T2:=int(diff(int(diff(16*T1,z),z),cz),cz):
S3:=S2:T3:=(T2/n/(n^2-1)):
k:=I:ck:=-I: M:=k*ck:
> f2:=a: g3:= 0: G3:= 0: H4:= 0: h4:= 0: g001:= 0: H101:= 0: h101:=
0: F01:= -f2: F2:= 1/12*r^2: f01:= 1/12*r^2: G11:= -2*r*f2: g11:=
-1/6*r^3: H21:= -2*r^2*f2: h21:= -1/6*r^4: h02:= -2*r^2*f2: G001:=
r*f2: H02:= -1/6*r^4:
> w2:=u2+I*z*cz: cw2:=u2-I*z*cz:
w3:=u3+I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):
> f:=
r*ck*z+
(F2+I*f2)*z^2+(F01+I*f01)*w2+
(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3:

cf:=
r*k*cz+

```



```
(F2-I*f2)*cz^2+(F01-I*f01)*cw2+
(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3:
```

```
g:=
r^2*M*w2+
(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:
```

```
cg:=
r^2*M*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:
```

```
h:=
r^3*M^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:
```

```
ch:=
r^3*M^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:
```

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
```

```
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (4,5)-jet.**

```
> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):
```

```
P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
```

```
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
```

```
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
```

```
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):
```

```
Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
```

```
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
```

```

Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz)))
:
Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write 26 equations.**

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

```

```

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

```

> sol:=solve(EQ):

```

```

> sol[2]:

```

$$\{H12 = \frac{1}{3} r^3 a, f3 = \frac{1}{144} \frac{3 n^2 r^4 - 144 a^2 + 2 r^4}{r}, g2I = \frac{1}{2} r^2 a,$$

$$g02 = -\frac{1}{16} n^2 r^4 + \frac{11}{144} r^4 - 4 a^2, F3 = -\frac{1}{6} r a, F00I = -\frac{1}{288} \frac{9 n^2 r^4 + 144 a^2 - 2 r^4}{r},$$

$$C1 = \frac{1}{24} \frac{3n^2 r^4 + 144a^2 - 2r^4}{r^2}, G21 = -\frac{1}{36} r^4 + 2a^2, F11 = 0,$$

$$H201 = -\frac{1}{144} r(9n^2 r^4 + 144a^2 - 2r^4), h31 = \frac{2}{3} r^3 a, n = n, g101 = -\frac{1}{6} r^2 a,$$

$$G101 = -\frac{1}{16} n^2 r^4 - a^2 + \frac{1}{72} r^4, G02 = -\frac{1}{6} r^2 a, H011 = -\frac{1}{3} r^3 a, h12 = 2ra^2 + \frac{1}{24} r^5,$$

$$a = a, h201 = -\frac{1}{6} r^3 a, f001 = \frac{1}{12} ra, H31 = 4ra^2 - \frac{1}{36} r^5, f11 = -\frac{1}{3} ra, C2 = 0,$$

$$G4 = 0, g4 = 0, H5 = 0, h5 = 0, h011 = 0, r = r\}$$

> CC1:=1/24\*(3\*r^4\*n^2+144\*a^2-2\*r^4)/(r^2):

> solve(CC1=0, a);

$$\frac{1}{12} \sqrt{-3n^2 + 2r^2}, -\frac{1}{12} \sqrt{-3n^2 + 2r^2}$$

### Third Step: (5,6) - normalization ----- (4,5,6) jet.

> restart:

readlib(mtaylor):

C1:=0: C2:=0: a:=p\*r^2:

x:=(z+cz)/2:y:=(z-cz)/(2\*I):

S:=exp(x)\*cos(n\*y):

T:=exp(x)\*sin(n\*y):

S1:=mtaylor(S,[z,cz],8): T1:=mtaylor(T,[z,cz],9):

S2:=int(diff(int(diff(4\*S1/(1-n^2),z),z),cz),cz):

T2:=int(diff(int(diff(16\*T1,z),z),cz),cz):

S3:=S2:T3:=(T2/n/(n^2-1)):

k:=I:ck:=-I: M:=k\*ck:

> f2:=a: g3:= 0: G3:= 0: H4:= 0: h4:= 0: g001:= 0: H101:= 0: h101:=

0: F01:= -f2: F2:= 1/12\*r^2: f01:= 1/12\*r^2: G11:= -2\*r\*f2: g11:=

-1/6\*r^3: H21:= -2\*r^2\*f2: h21:= -1/6\*r^4: h02:= -2\*r^2\*f2: G001:=

r\*f2: H02:= -1/6\*r^4:

H12:= 1/3\*r^3\*a: f001:= 1/12\*r\*a: f11:= -1/3\*r\*a: g21:= 1/2\*r^2\*a:

H31:= -1/36\*r^5+4\*r\*a^2: h12:= 2\*r\*a^2+1/24\*r^5: F11:= 0: f3:=

1/144\*(3\*r^4\*n^2-144\*a^2+2\*r^4)/r: g02:= -1/16\*r^4\*n^2-

4\*a^2+11/144\*r^4: F3:= -1/6\*r\*a: F001:= -1/288\*(9\*r^4\*n^2+144\*a^2-

2\*r^4)/r: h201:= -1/6\*r^3\*a: h31:= 2/3\*r^3\*a: H201:=

-1/144\*r\*(9\*r^4\*n^2+144\*a^2-2\*r^4): h011:= 0: G4:= 0: H5:= 0: h5:=

0: g4:= 0: H011:= -1/3\*r^3\*a: G02:= -1/6\*r^2\*a: G101:=

-1/16\*r^4\*n^2-a^2+1/72\*r^4: g101:= -1/6\*r^2\*a: G21:= 2\*a^2-

1/36\*r^4:

**Write a mapping**

> f:=

r\*ck\*z+

$(F2+I*f2)*z^2+(F01+I*f01)*w2+$   
 $(F3+I*f3)*z^3+(F11+I*f11)*z*w2+(F001+I*f001)*w3+$   
 $(F4+I*f4)*z^4+(F21+I*f21)*z^2*w2+(F02+I*f02)*w2^2+$   
 $(F101+I*f101)*z*w3:$

$cf:=$   
 $r*k*cz+$   
 $(F2-I*f2)*cz^2+(F01-I*f01)*cw2+$   
 $(F3-I*f3)*cz^3+(F11-I*f11)*cz*cw2+(F001-I*f001)*cw3+$   
 $(F4-I*f4)*cz^4+(F21-I*f21)*cz^2*cw2+(F02-I*f02)*cw2^2+(F101-I*f101)*cz*cw3:$

$g:=$   
 $r^2*M*w2+$   
 $(G3+I*g3)*z^3+(G11+I*g11)*z*w2+(G001+I*g001)*w3+$   
 $(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+$   
 $(G101+I*g101)*z*w3+$   
 $(G5+I*g5)*z^5+(G31+I*g31)*z^3*w2+(G12+I*g12)*z*w2^2+$   
 $(G201+I*g201)*z^2*w3+(G011+I*g011)*w2*w3:$

$cg:=$   
 $r^2*M*cw2+$   
 $(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+$   
 $(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-I*g101)*cz*cw3+$   
 $(G5-I*g5)*cz^5+(G31-I*g31)*cz^3*cw2+(G12-I*g12)*cz*cw2^2+(G201-I*g201)*cz^2*cw3+(G011-I*g011)*cw2*cw3:$

$h:=$   
 $r^3*M^2*w3+$   
 $(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+$   
 $(H101+I*h101)*z*w3+$   
 $(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+$   
 $(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3+$   
 $(H6+I*h6)*z^6+(H41+I*h41)*z^4*w2+(H22+I*h22)*z^2*w2^2+$   
 $(H03+I*h03)*w2^3+(H301+I*h301)*z^3*w3+(H111+I*h111)*z*w2*w3+$   
 $(H002+I*h002)*w3^2:$

$ch:=$   
 $r^3*M^2*cw3+$   
 $(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-I*h101)*cz*cw3+$   
 $(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3+$   
 $(H6-I*h6)*cz^6+(H41-I*h41)*cz^4*cw2+(H22-I*h22)*cz^2*cw2^2+(H03-I*h03)*cw2^3+(H301-I*h301)*cz^3*cw3+(H111-I*h111)*cz*cw2*cw3+(H002-I*h002)*cw3^2:$

```
> w2:=u2+I*(z*cz+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+(E1-
I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
cw2:=u2-I*(z*cz+(D1+I*D2)*z^4*cz+(E1+I*E2)*z^3*cz^2+(E1-
I*E2)*cz^3*z^2+(D1-I*D2)*cz^4*z):
```

```
w3:=u3+I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):
cw3:=u3-I*(z^2*cz+cz^2*z+(C1+I*C2)*z^3*cz^2+(C1-I*C2)*cz^3*z^2):
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (5,6)-jet.**

```
>
P[5,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,z)))):
P[4,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,z,cz)))):
P[3,2,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,cz,cz)))):
P[3,0,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z,u2)))):
P[2,1,1,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,u2)))):
P[1,0,2,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2,u2)))):
P[2,0,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u3)))):
P[1,1,0,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u3)))):
P[0,0,1,1]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u2,u3)))):
```

```
P1[5,0,0,0]:=subs(I=0,P[5,0,0,0]):
P2[5,0,0,0]:=normal(expand((P[5,0,0,0]-P1[5,0,0,0])*I)):
P1[4,1,0,0]:=subs(I=0,P[4,1,0,0]):
P2[4,1,0,0]:=normal(expand((P[4,1,0,0]-P1[4,1,0,0])*I)):
P1[3,2,0,0]:=subs(I=0,P[3,2,0,0]):
P2[3,2,0,0]:=normal(expand((P[3,2,0,0]-P1[3,2,0,0])*I)):
P1[3,0,1,0]:=subs(I=0,P[3,0,1,0]):
P2[3,0,1,0]:=normal(expand((P[3,0,1,0]-P1[3,0,1,0])*I)):
P1[2,1,1,0]:=subs(I=0,P[2,1,1,0]):
P2[2,1,1,0]:=normal(expand((P[2,1,1,0]-P1[2,1,1,0])*I)):
P1[1,0,2,0]:=subs(I=0,P[1,0,2,0]):
P2[1,0,2,0]:=normal(expand((P[1,0,2,0]-P1[1,0,2,0])*I)):
P1[2,0,0,1]:=subs(I=0,P[2,0,0,1]):
P2[2,0,0,1]:=normal(expand((P[2,0,0,1]-P1[2,0,0,1])*I)):
```

```
Q[6,0,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z,z)))):
Q[5,1,0,0]:=normal(expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z,z)))):
```

```

, z, cz))) :
Q[4, 2, 0, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, z, z,
, cz, cz)))) :
Q[3, 3, 0, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, z, c
z, cz, cz)))) :
Q[4, 0, 1, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, z, z
, u2)))) :
Q[3, 1, 1, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, z, c
z, u2)))) :
Q[2, 2, 1, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, cz,
cz, u2)))) :
Q[2, 0, 2, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, u2,
u2)))) :
Q[1, 1, 2, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, cz, u2
, u2)))) :
Q[0, 0, 3, 0] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, u2, u2, u
2)))) :
Q[3, 0, 0, 1] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, z, u
3)))) :
Q[2, 1, 0, 1] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, z, cz,
u3)))) :
Q[1, 0, 1, 1] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, z, u2, u3
)))) :
Q[0, 0, 0, 2] := normal (expand (subs ([z=0, cz=0, u2=0, u3=0], diff (e3, u3, u3)
)))) :

```

```

Q1[6, 0, 0, 0] := subs (I=0, Q[6, 0, 0, 0]) :
Q2[6, 0, 0, 0] := normal (expand ((Q[6, 0, 0, 0]-Q1[6, 0, 0, 0])*I)) :
Q1[5, 1, 0, 0] := subs (I=0, Q[5, 1, 0, 0]) :
Q2[5, 1, 0, 0] := normal (expand ((Q[5, 1, 0, 0]-Q1[5, 1, 0, 0])*I)) :
Q1[4, 2, 0, 0] := subs (I=0, Q[4, 2, 0, 0]) :
Q2[4, 2, 0, 0] := normal (expand ((Q[4, 2, 0, 0]-Q1[4, 2, 0, 0])*I)) :

```

```

Q1[4, 0, 1, 0] := subs (I=0, Q[4, 0, 1, 0]) :
Q2[4, 0, 1, 0] := normal (expand ((Q[4, 0, 1, 0]-Q1[4, 0, 1, 0])*I)) :
Q1[3, 1, 1, 0] := subs (I=0, Q[3, 1, 1, 0]) :
Q2[3, 1, 1, 0] := normal (expand ((Q[3, 1, 1, 0]-Q1[3, 1, 1, 0])*I)) :

```

```

Q1[2, 0, 2, 0] := subs (I=0, Q[2, 0, 2, 0]) :
Q2[2, 0, 2, 0] := normal (expand ((Q[2, 0, 2, 0]-Q1[2, 0, 2, 0])*I)) :

```

```

Q1[3, 0, 0, 1] := subs (I=0, Q[3, 0, 0, 1]) :
Q2[3, 0, 0, 1] := normal (expand ((Q[3, 0, 0, 1]-Q1[3, 0, 0, 1])*I)) :
Q1[2, 1, 0, 1] := subs (I=0, Q[2, 1, 0, 1]) :
Q2[2, 1, 0, 1] := normal (expand ((Q[2, 1, 0, 1]-Q1[2, 1, 0, 1])*I)) :
Q1[1, 0, 1, 1] := subs (I=0, Q[1, 0, 1, 1]) :
Q2[1, 0, 1, 1] := normal (expand ((Q[1, 0, 1, 1]-Q1[1, 0, 1, 1])*I)) :

```

**Write 39 equations.**

> EQ := {

P1[5,0,0,0]=0, P2[5,0,0,0]=0,  
P1[4,1,0,0]=0, P2[4,1,0,0]=0,  
P1[3,2,0,0]=0, P2[3,2,0,0]=0,  
P1[3,0,1,0]=0, P2[3,0,1,0]=0,  
P1[2,1,1,0]=0, P2[2,1,1,0]=0,  
P1[1,0,2,0]=0, P2[1,0,2,0]=0,  
P1[2,0,0,1]=0, P2[2,0,0,1]=0,  
P[1,1,0,1]=0,  
P[0,0,1,1]=0,

Q1[6,0,0,0]=0, Q2[6,0,0,0]=0,  
Q1[5,1,0,0]=0, Q2[5,1,0,0]=0,  
Q1[4,2,0,0]=0, Q2[4,2,0,0]=0,  
Q[3,3,0,0]=0,  
Q1[4,0,1,0]=0, Q2[4,0,1,0]=0,  
Q1[3,1,1,0]=0, Q2[3,1,1,0]=0,  
Q[2,2,1,0]=0,  
Q1[2,0,2,0]=0, Q2[2,0,2,0]=0,  
Q[1,1,2,0]=0,  
Q[0,0,3,0]=0,  
Q1[3,0,0,1]=0, Q2[3,0,0,1]=0,  
Q1[2,1,0,1]=0, Q2[2,1,0,1]=0,  
Q1[1,0,1,1]=0, Q2[1,0,1,1]=0,  
Q[0,0,0,2]=0  
}:

> sol := solve(EQ);

sol := {H6 = 0, h6 = 0, G201 = 0, h22 = 0, h111 = 0, p = p, g201 = 0, g31 = 0, G31 = 0,  
H03 = 0, H22 = 0, H41 = 0, h301 = 0, G5 = 0, g5 = 0, h002 = 0, h41 = 0, H301 = 0,  
H111 = 0, G011 = 0, H002 = 0, h03 = 0, g011 = 0, g12 = 0, n = n, F4 = F4, F21 = F21,  
F02 = F02, f21 = f21, f02 = f02, f4 = f4, F101 = F101, f101 = f101, D1 = D1, D2 = D2,

$$E1 = E1, E2 = E2, G12 = 0, r = 0\}, \{H6 = 0, h6 = 0, H22 = \frac{167}{16416} r^6, E2 = -\frac{7}{1368} r^3,$$

$$F4 = -\frac{5}{2736} r^4, G201 = 0, F101 = 0, f101 = -\frac{11}{513} r^4, f02 = 0, F21 = 0, f21 = \frac{1}{4104} r^4,$$

$$h22 = 0, h111 = 0, F02 = -\frac{1}{8208} r^4, h301 = \frac{13}{1368} r^6, E1 = 0, G31 = 0, H03 = 0,$$

$$g201 = \frac{13}{1368} r^5, g31 = \frac{29}{2052} r^5, H41 = 0, G5 = 0, g5 = 0, h002 = 0, H301 = 0,$$

$$h41 = \frac{29}{2052} r^6, H111 = \frac{13}{1368} r^6, D1 = 0, G011 = \frac{157}{8208} r^5, p = 0,$$

$$n = \frac{1}{3} \text{RootOf}(19\_Z^2 - 194), r = r, H002 = \frac{49}{2736} r^6, h03 = -\frac{1}{864} r^6, g011 = 0, g12 = 0,$$

$$f4 = 0, G12 = \frac{53}{16416} r^5, D2 = -\frac{35}{8208} r^3 \}$$

>

The values  $r=0$  and  $p=0$  are not under consideration. So, it is not the cubic.

## Type Vb.

First step: (3,4)-normalization -----

### (2,3,4) jet

```
> restart:readlib(mttaylor):
x:=(z+cz)/2:y:=(z-cz)/(2*I):A:=0:
S:=exp(x)*y*sin(y):T:=exp(x)*y*cos(y):
S1:=mtaylor(2*S,[z,cz],8):T1:=mtaylor(8*T,[z,cz],8):
S2:=int(diff(int(diff(S1,z),z),cz),cz):
T2:=int(diff(int(diff(T1,z),z),cz),cz):
S3:=subs([z=I*z,cz=-I*cz],S2):
T3:=subs([z=I*z,cz=-I*cz],-T2):
```

#### Result check.

```
> S[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z)))));
S[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,cz)))));
S[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(S3,z,z)))));
print('xxxxxxxxxxxxxxxxxxxxxxxxxxxx');
T[1,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z)))));
T[1,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,cz)))));
T[2,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z)))));
T[2,1,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,cz)))))/2;
T[3,0,0,0]:=simplify(expand(subs([z=0,cz=0],diff(T3,z,z,z)))));
```

$$S_{1,0,0,0} := 0$$

$$S_{1,1,0,0} := 1$$

$$S_{2,0,0,0} := 0$$

xxxxxxxxxxxxxxxxxxxxxxxx

$$T_{1,0,0,0} := 0$$



$$T_{1,1,0,0} := 0$$

$$T_{2,0,0,0} := 0$$

$$T_{2,1,0,0} := 1$$

$$T_{3,0,0,0} := 0$$

>

**Introduce the notations.**

> k:=1:ck:=1: m:=1:

**Introduce new variables**

> w2:=u2+I\*z\*cz: cw2:=u2-I\*z\*cz:

w3:=u3+I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):

cw3:=u3-I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2):

**Write a mapping**

> f:=

r\*ck\*z+

(F2+I\*f2)\*z^2+(F01+I\*f01)\*w2:

cf:=

r\*k\*cz+

(F2-I\*f2)\*cz^2+(F01-I\*f01)\*cw2:

g:=

r^2\*m\*w2+

(G3+I\*g3)\*z^3+(G11+I\*g11)\*z\*w2+(G001+I\*g001)\*w3:

cg:=

r^2\*m\*cw2+

(G3-I\*g3)\*cz^3+(G11-I\*g11)\*cz\*cw2+(G001-I\*g001)\*cw3:

h:=

r^3\*m^2\*w3+

(H4+I\*h4)\*z^4+(H21+I\*h21)\*z^2\*w2+(H02+I\*h02)\*w2^2+

(H101+I\*h101)\*z\*w3:

ch:=

r^3\*m^2\*cw3+

(H4-I\*h4)\*cz^4+(H21-I\*h21)\*cz^2\*cw2+(H02-I\*h02)\*cw2^2+(H101-

I\*h101)\*cz\*cw3:

**Write the relations for the mapping.**

> e2:=subs([z=f,cz=cf],2\*S3)+I\*(g-cg):

```
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (3,4)-jet.**

```
> P[3,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z))):
P[2,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz))):
P[1,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u2))):
P[0,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,u3))):
```

```
P1[3,0,0,0]:=subs(I=0,P[3,0,0,0]): P2[3,0,0,0]:=expand((P[3,0,0,0]-
P1[3,0,0,0])*I):
P1[2,1,0,0]:=subs(I=0,P[2,1,0,0]): P2[2,1,0,0]:=expand((P[2,1,0,0]-
P1[2,1,0,0])*I):
P1[1,0,1,0]:=subs(I=0,P[1,0,1,0]): P2[1,0,1,0]:=expand((P[1,0,1,0]-
P1[1,0,1,0])*I):
P1[0,0,0,1]:=subs(I=0,P[0,0,0,1]): P2[0,0,0,1]:=expand((P[0,0,0,1]-
P1[0,0,0,1])*I):
```

```
Q[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z))):
Q[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz))):
Q[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,cz))):
Q[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u2))):
Q[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u2))):
Q[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u3))):
```

```
Q1[4,0,0,0]:=subs(I=0,Q[4,0,0,0]): Q2[4,0,0,0]:=expand((Q[4,0,0,0]-
Q1[4,0,0,0])*I):
Q1[3,1,0,0]:=subs(I=0,Q[3,1,0,0]): Q2[3,1,0,0]:=expand((Q[3,1,0,0]-
Q1[3,1,0,0])*I):
Q1[2,0,1,0]:=subs(I=0,Q[2,0,1,0]): Q2[2,0,1,0]:=expand((Q[2,0,1,0]-
Q1[2,0,1,0])*I):
Q1[1,0,0,1]:=subs(I=0,Q[1,0,0,1]): Q2[1,0,0,1]:=expand((Q[1,0,0,1]-
Q1[1,0,0,1])*I):
```

**Write 17 equations.**

```
> EQ:={
P1[3,0,0,0]=0,P2[3,0,0,0]=0,
P1[2,1,0,0]=0,P2[2,1,0,0]=0,
P1[1,0,1,0]=0,P2[1,0,1,0]=0,
P[0,0,0,1]=0,
```

```
Q1[4,0,0,0]=0,Q2[4,0,0,0]=0,
Q1[3,1,0,0]=0,Q2[3,1,0,0]=0,
Q[2,2,0,0]=0,
Q1[2,0,1,0]=0,Q2[2,0,1,0]=0,
Q[1,1,1,0]=0,
Q1[1,0,0,1]=0,Q2[1,0,0,1]=0
}:

```

**Solve the system.**

```
> solve(EQ);
```

$\{H101 = 0, h101 = 0, g001 = 0, H4 = 0, h4 = 0, G3 = 0, g3 = 0, h21 = 0, H21 = 0, h02 = 0,$   
 $r = 0, G001 = 0, g11 = 0, G11 = 0, H02 = 0, F2 = F2, f2 = f2, F01 = F01, f01 = f01\}, \{$   
 $H21 = 2 r^2 f01, H101 = 0, h101 = 0, g001 = 0, G11 = 2 r f01, H4 = 0, h4 = 0, G3 = 0,$   
 $g3 = 0, h21 = \frac{1}{6} r^4, F01 = \frac{1}{12} r^2, h02 = 2 r^2 f01, f2 = -\frac{1}{12} r^2, F2 = f01, G001 = -r f01,$   
 $g11 = \frac{1}{6} r^3, H02 = \frac{1}{6} r^4, f01 = f01, r = r\}$

>

## Second Step: (4,5)-normalization ----- (3,4,5) jet

> restart: readlib(mtaylor):A:=0: B:=0:

**Write the results of the previous step.**

> f01:= a: g001:= 0: G3:= 0: g3:= 0: H4:= 0: h4:= 0: H101:= 0:  
 h101:= 0: G11:= 2\*r\*f01: H21:= 2\*r^2\*f01: F2:= f01: f2:= -1/12\*r^2:  
 F01:= 1/12\*r^2: g11:= 1/6\*r^3: G001:= -r\*f01: h21:= 1/6\*r^4: H02:=  
 1/6\*r^4: h02:= 2\*r^2\*f01:

> x:=(z+cz)/2:y:=(z-cz)/(2\*I):

S:=exp(x)\*y\*sin(y):T:=exp(x)\*y\*cos(y):

S1:=mtaylor(2\*S,[z,cz],8):T1:=mtaylor(8\*T,[z,cz],8):

S2:=int(diff(int(diff(S1,z),z),cz),cz):

T2:=int(diff(int(diff(T1,z),z),cz),cz):

S3:=subs([z=I\*z,cz=-I\*cz],S2):

T3:=subs([z=I\*z,cz=-I\*cz],-T2):

k:=1:ck:=1: m:=1:

> w2:=u2+I\*(z\*cz+B\*z^2\*cz^2): cw2:=u2-I\*(z\*cz+B\*z^2\*cz^2):

w3:=u3+I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2+(C1+I\*C2)\*z^3\*cz^2+(C1-I\*C2)\*cz^3\*z^2):

cw3:=u3-I\*(z^2\*cz+cz^2\*z+A\*z^2\*cz^2+(C1+I\*C2)\*z^3\*cz^2+(C1-I\*C2)\*cz^3\*z^2):

**Write a mapping**

> f:=

r\*ck\*z+

(F2+I\*f2)\*z^2+(F01+I\*f01)\*w2+

(F3+I\*f3)\*z^3+(F11+I\*f11)\*z\*w2+(F001+I\*f001)\*w3:

cf:=

r\*k\*cz+

(F2-I\*f2)\*cz^2+(F01-I\*f01)\*cw2+

(F3-I\*f3)\*cz^3+(F11-I\*f11)\*cz\*cw2+(F001-I\*f001)\*cw3:

g:=

r^2\*m\*w2+

(G3+I\*g3)\*z^3+(G11+I\*g11)\*z\*w2+(G001+I\*g001)\*w3+

```
(G4+I*g4)*z^4+(G21+I*g21)*z^2*w2+(G02+I*g02)*w2^2+
(G101+I*g101)*z*w3:
```

```
cg:=
r^2*m*cw2+
(G3-I*g3)*cz^3+(G11-I*g11)*cz*cw2+(G001-I*g001)*cw3+
(G4-I*g4)*cz^4+(G21-I*g21)*cz^2*cw2+(G02-I*g02)*cw2^2+(G101-
I*g101)*cz*cw3:
```

```
h:=
r^3*m^2*w3+
(H4+I*h4)*z^4+(H21+I*h21)*z^2*w2+(H02+I*h02)*w2^2+
(H101+I*h101)*z*w3+
(H5+I*h5)*z^5+(H31+I*h31)*z^3*w2+(H12+I*h12)*z*w2^2+
(H201+I*h201)*z^2*w3+(H011+I*h011)*w2*w3:
```

```
ch:=
r^3*m^2*cw3+
(H4-I*h4)*cz^4+(H21-I*h21)*cz^2*cw2+(H02-I*h02)*cw2^2+(H101-
I*h101)*cz*cw3+
(H5-I*h5)*cz^5+(H31-I*h31)*cz^3*cw2+(H12-I*h12)*cz*cw2^2+(H201-
I*h201)*cz^2*cw3+(H011-I*h011)*cw2*cw3:
```

**Write the relations for the mapping.**

```
> e2:=subs([z=f,cz=cf],2*S3)+I*(g-cg):
e3:=subs([z=f,cz=cf],2*T3)+I*(h-ch):
```

**Write the coefficients of the (4,5)-jet.**

```
> P[4,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,z))):
P[3,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,z,cz))):
P[2,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,cz,cz))):
P[2,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,z,u2))):
P[1,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,cz,u2))):
P[1,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e2,z,u3))):
```

```
P1[4,0,0,0]:=subs(I=0,P[4,0,0,0]): P2[4,0,0,0]:=expand((P[4,0,0,0]-
P1[4,0,0,0])*I):
P1[3,1,0,0]:=subs(I=0,P[3,1,0,0]): P2[3,1,0,0]:=expand((P[3,1,0,0]-
P1[3,1,0,0])*I):
P1[2,0,1,0]:=subs(I=0,P[2,0,1,0]): P2[2,0,1,0]:=expand((P[2,0,1,0]-
P1[2,0,1,0])*I):
P1[1,0,0,1]:=subs(I=0,P[1,0,0,1]): P2[1,0,0,1]:=expand((P[1,0,0,1]-
P1[1,0,0,1])*I):
```

```
Q[5,0,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,z))):
Q[4,1,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,z,cz))):
Q[3,2,0,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,cz,cz))):
:
```

```

Q[3,0,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,z,u2))):
Q[2,1,1,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,cz,u2))):
Q[1,0,2,0]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,u2,u2))):
Q[2,0,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,z,u3))):
Q[1,1,0,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,z,cz,u3))):
Q[0,0,1,1]:=expand(subs([z=0,cz=0,u2=0,u3=0],diff(e3,u2,u3))):

```

```

Q1[5,0,0,0]:=subs(I=0,Q[5,0,0,0]): Q2[5,0,0,0]:=expand((Q[5,0,0,0]-
Q1[5,0,0,0])*I):
Q1[4,1,0,0]:=subs(I=0,Q[4,1,0,0]): Q2[4,1,0,0]:=expand((Q[4,1,0,0]-
Q1[4,1,0,0])*I):
Q1[3,2,0,0]:=subs(I=0,Q[3,2,0,0]): Q2[3,2,0,0]:=expand((Q[3,2,0,0]-
Q1[3,2,0,0])*I):
Q1[3,0,1,0]:=subs(I=0,Q[3,0,1,0]): Q2[3,0,1,0]:=expand((Q[3,0,1,0]-
Q1[3,0,1,0])*I):
Q1[2,1,1,0]:=subs(I=0,Q[2,1,1,0]): Q2[2,1,1,0]:=expand((Q[2,1,1,0]-
Q1[2,1,1,0])*I):
Q1[1,0,2,0]:=subs(I=0,Q[1,0,2,0]): Q2[1,0,2,0]:=expand((Q[1,0,2,0]-
Q1[1,0,2,0])*I):
Q1[2,0,0,1]:=subs(I=0,Q[2,0,0,1]): Q2[2,0,0,1]:=expand((Q[2,0,0,1]-
Q1[2,0,0,1])*I):

```

**Write the equations and the variables.**

```

> EQ:={
P1[4,0,0,0]=0,P2[4,0,0,0]=0,
P1[3,1,0,0]=0,P2[3,1,0,0]=0,
P[2,2,0,0]=0,
P1[2,0,1,0]=0,P2[2,0,1,0]=0,
P[1,1,1,0]=0,
P1[1,0,0,1]=0,P2[1,0,0,1]=0,

```

```

Q1[5,0,0,0]=0,Q2[5,0,0,0]=0,
Q1[4,1,0,0]=0,Q2[4,1,0,0]=0,
Q1[3,2,0,0]=0,Q2[3,2,0,0]=0,
Q1[3,0,1,0]=0,Q2[3,0,1,0]=0,
Q1[2,1,1,0]=0,Q2[2,1,1,0]=0,
Q1[1,0,2,0]=0,Q2[1,0,2,0]=0,
Q1[2,0,0,1]=0,Q2[2,0,0,1]=0,
Q[1,1,0,1]=0,
Q[0,0,1,1]=0
}:

```

**Number of equations - 26.**

```

> solve(EQ)[2];

```

$$\{r = r, G101 = -a^2 - \frac{7}{144}r^4, H12 = \frac{1}{3}r^3 a, g21 = \frac{1}{2}r^2 a, H011 = -\frac{1}{3}r^3 a, h5 = 0, h011 = 0, \\ g4 = 0, H5 = 0, G4 = 0, g101 = -\frac{1}{6}r^2 a, h12 = \frac{1}{24}r^5 + 2r a^2, H31 = -\frac{1}{36}r^5 + 4r a^2,$$

$$G21 = 2 a^2 - \frac{1}{36} r^4, a = a, f001 = -\frac{1}{288} \frac{144 a^2 + 7 r^4}{r}, CI = \frac{1}{24} \frac{144 a^2 + r^4}{r^2},$$

$$F3 = \frac{1}{144} \frac{144 a^2 - 5 r^4}{r}, f3 = -\frac{1}{6} r a, g02 = -4 a^2 + \frac{1}{72} r^4, f11 = 0, h31 = \frac{2}{3} r^3 a, C2 = 0,$$

$$F11 = \frac{1}{3} r a, F001 = -\frac{1}{12} r a, h201 = -\frac{1}{6} r^3 a, H201 = -\frac{1}{144} r (144 a^2 + 7 r^4),$$

$$G02 = -\frac{1}{6} r^2 a \}$$

> CC1 := 1/24\*(r^4+144\*a^2)/(r^2);

$$CCI := \frac{1}{24} \frac{144 a^2 + r^4}{r^2}$$

$$CCI := \frac{1}{24} \frac{144 a^2 + r^4}{r^2}$$

**Then it is not the cubic.**